EMORY MATHEMATICS DIRECTED READING PROGRAM

Program description

The Emory Math Directed Reading Program (DRP) is a graduate student-run program aiming to pair undergraduate students with graduate student mentors to read and learn material that is not typically offered in a traditional course setting. Undergraduate students are expected to work mostly independently to read the text and attempt exercises, then meet regularly with their graduate student mentor to discuss the material.

Steering Committee

If you have any questions or concerns, or suggestions for future DRP topics, please reach out to Ylli Andoni at ylli.andoni@emory.edu, Guangqiu Liang at guangqiu.liang@emory.edu, Mitchell Scott at mitchell.scott@emory.edu, or Akash Sureshkumar at akash.sureshkumar@emory.edu.

New Proposed Topics and Descriptions

Students specify a topic(s) of interest when applying to the program, in order to be matched with an appropriate graduate mentor. While the other document has a list of selected topics that have been done before, below are topics that are newly proposed by graduate students, and they are very eager to do a project on. If a topic sounds interesting, please see the next page for more detailed sample descriptions.

Algebra.

- Coding Theory and Algebraic Geometry

Analysis and Geometry.

- Variational Calculus

Applied and Computational Math.

- Deep Generative Modeling
- Uncertainty Quantification for Inverse Problems

Discrete Math and Logic.

- Information Theory
- Philosophy of Mathematics
- History of Mathematics

Course name: Coding Theory and Algebraic Geometry
Text: *Codes and Curves*, by Judy Walker
Prerequisites: Abstract Algebra I (Math 421)

Description: Whenever data is transmitted across a channel, errors are likely to occur. It is the goal of coding theory to find efficient ways of encoding the data so that these errors can be detected, or even corrected. Normally this is done using group theory or discrete math; however, we plan to use different course of action – algebraic geometric codes. The goal of this course is to introduce you to some of the basics of coding theory, algebraic geometry, and algebraic geometric codes.

If you have no idea what codes are, we could think of the serial number of a dollar bill or the International Standard Book Number (ISBN), which can uniquely identify the dollar bill or book you have with some redundancies to make counterfeiting harder or correct if the ISBM is smudged, respectively.

Course name: Variational Calculus
Text: *Calculus of Variation*, by I.M. Gelfand and S.V. Fomin
Prerequisites:

Description: You might have heard about the "great circle route" before. This is when planes fly very north over Antartica, but why? The map looks like there should be a shorter path. However, on the surface of the sphere, it can be proven that the arc IS the shortest path. The technique needed to prove this is the calculus of variations. This DRP will first introduce the calculus of variations, which studies *functionals*, where the input is a function, and the output is a real valued scalar. From this, we will extend calculus to these functionals, take derivatives, and find the minimum such function that maintinas such desired properties. There are many physical examples that will be presented.

Course name: Deep Generative Modeling
Text: *Deep Generative Modeling*, by Jakub M. Tomczak
Prerequisites: Undergraduate Probability (Math 361)

Description: I just saw is a photo of George Washington riding a dinosaur across the surface of Mars, but how did they get that? Photography wasn't invented yet, dinosaurs and George Washington didn't live during the same time, and Mars is far away. This is an extreme example of generative modeling, where we assume that data has an underlying distribution. Combining supervised learning and unsupervised learning, the resulting paradigm is called deep generative modeling, which utilizes the generative perspective on perceiving the surrounding world. It assumes that each phenomenon is driven by an underlying generative process that defines a joint distribution over random variables and their stochastic interactions, i.e., how events occur and in what order. The ultimate aim of the course is to outline the most important techniques in deep generative modeling and, eventually, enable readers to formulate new models and implement them.

Course name: Uncertainty Quantification for Inverse Problems
Text: *Discrete Inverse Problems: Insight and Algorithms*, by Per Christian Hansen
Prerequisites: Numerical Analysis (Math 315)

Description: A forward problem would be given a set of all these parameters, find out the final state of the model. This seems rather straight forward as you can perform a simple matrix multiplication or function evaluation depending on the system. Conversely, an inverse problem is given the final output of the system, can you determine the parameters that caused this system to evolve like this. It sounds much harder because it is. An added issue with inverse problems is since the true parameters are unknown, how do you figure out how close your guess is to the true solution? This is where uncertainty quantification comes in. It allows you to measure how far you are away from the unknown solution using the techniques of inverse problems.

Course name: Information Theory
Text: *Elements of Information Theory*, by Thomas Cover and Joy Thomas
Prerequisites: Undergraduate Probability (Math 361)

Description: Have you ever tried to download a large file, and it takes forever? Do you ever wonder if there is some sort of cosmic speed at which data is traveling, and maybe the file you are downloading is as fast as possible without breaking the laws of physcis. In this DRP, we discuss the theory of information. We define data "entropy" as the minimum descriptive complexity of a random variable,and we answer the question "does entropy always increase?" Then we discuss how information theory relates to probability, communiciation, and statistics. There will be applications to gambling, data compression, and more.

Course name: Philosophy of Mathematics
Text: *Thinking about Mathematics*, by Stewart Shapiro
Prerequisites: None

Description: You have taken a class on proofs before, where the professor asked you to prove $\sqrt{2}$ is irrational, and you followed some hints and figured it out. The professor gave you a perfect score. But how do you know it is correct? How do we know anything in math? What is a number? What're the axioms? What is mathematical methodology? In this DRP, we try to link math, a cut-and-dried discipline, and philosophy, a very broad and all encompassing topic. We discuss how we think about math, the history, perspective, how philosophers have thought about math, and where math is in the modern scene. Finally, at the end of the DRP, we will answer the age-old question: is math discovered or invented? ;P

Course name: History of Mathematics
Text: *Mathematical thought from Mesopotamia to the birth of Calculus, Vol.1*, by Morris Kline
Prerequisites: None

Description: Who invented calculus? Was it Newton? or was it Leibnitz? Who cares! While this question is often thought of as the quintessential history of math question, math was already around for eons. In this DRP, we will investigate math from the very beginning, stopping before calculus, because you have already heard about the European mathematicians of the last few centuries. We will start in Mesopotamia and ancient Egypt where geomtery first started. We will traverse through the Greeks, the Chinese, the Indians, and see how math is truly universal, as countless cultures have been investigating it from the very beginning.