Algebra Seminar

Mock theta functions, q-series, combinatorial probability, and percolation models

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Abstract: In probabilistic percolation models (such as the Ising model for ferromagnetism), the sites of a lattice are randomly set to be active or inactive through an independent Bernoulli process. The system then evolves through simple deterministic growth rules that allow active sites to "flow" to inactive neighbors; for example, an inactive site may become active if at least two of its neighbors are active. These systems are very interesting due to the fact that they possess critical behavior, which means that there is a sharp density cutoff below which the system experiences very little growth, and above which almost every site is likely to eventually become active. Furthermore, as the size of the lattice grows to infinity, the critical probability approaches zero, and the critical window follows a finite-size (threshold) scaling relationship in this limit.
Amazingly, this limiting process also leads to the surprise appearance of functions of combinatorial and number theoretic interest, including Ramanujan's famous mock theta functions as well as the generating functions for partitions without sequences. The interplay between combinatorial probability, hypergeometric q-series, and automorphic forms leads to new proofs of conjectures posed by both probabilists and number theorists, involving slow convergence results for metastability thresholds and improved asymptotics for the generating functions of partitions without k-sequences.

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