Analysis and Differential Geometry
Dissertation Defense

On the Near-Field Reflector Problem and Optimal Transport

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Abstract: We will discuss the connection between optimal transport and the near-field reflector problem. In the near-field reflector problem, we are given a point source of light with some radiation intensity and a target set at a finite distance. The design problem consists of constructing a reflector that reflects the rays emitted by the source so that a given irradiance distribution on the target is produced.

In recent years, the optimal transport framework has been applied successfully to various open problems in the design of free-form lenses and reflectors. In this talk, we will investigate the near-field problem in this context. In particular, we will see that the notion of a weak solution to the near-field problem as an envelope of ellipsoids of revolution leads to a generalized Legendre transform. Aside from some interesting properties of this transform, it also gives rise to a variational problem that is naturally associated with the near-field reflector problem. Furthermore, the resulting variational problem resembles a generalized optimal transport problem and exhibits interesting analogies to other optimal transport problems arising in optical design and geometry, particularly to the far-field reflector problem. However, we will see that for the near-field problem the solutions to the associated variational problem do not solve the reflector problem in general. This is in strong contrast to the problems that have been studied previously in the optimal transport framework. Interestingly, we can still establish a connection between the solutions to the near-field problem and the variational problem. In particular, for discrete target sets we will present an approximation result, which shows that under a suitable choice of the admissible set the variational solution produces an irradiance distribution arbitrarily close to the prescribed irradiance distribution.

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