Dissertation Defense

Polynomials non-negative on non-compact subsets of the plane

Ha Nguyen
Emory University

Abstract: In 1991, Schmüdgen proved that if a polynomial f in n variables with real coeffcients such that $f > 0$ on a compact set K_S, then there always exists an algebraic expression showing that f is positive on K_S. Then in 1999, Scheiderer showed that if K_S is not compact and its dimension is 3 or more, there is no analogue of Schmüdgen's Theorem. However, in the noncompact two-dimensional case, very little is known about when every f positive or nonnegative on a noncompact set $K_S \subseteq \mathbb{R}^2$ has an algebraic expression proving that f is nonnegative on K_S. Recently, M. Marshall answered a long-standing question in real algebraic geometry by showing that if $f(x, y) \in \mathbb{R}[x, y]$ and $f(x, y) \geq 0$ on the strip $[0, 1] \times \mathbb{R}$, then f has a representation $f = \sigma_0 + \sigma_1 x(1 - x)$, where $\sigma_0, \sigma_1 \in \mathbb{R}[x, y]$ are sums of squares.

In this talk I will give some background to Marshall's result, which goes back to Hilbert's 17th problem, and our generalizations to other noncompact basic closed semialgebraic sets of $\mathbb{R}^2$ which are contained in strip. We also give some negative results.

Friday, April 2, 2010, 4:00 pm
Mathematics and Science Center: W201

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Emory University