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Merkurjev's Norm Principle

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Abstract: A classical result of Scharlau gives that $N_{L/k}(G_L(q_L)) \subset G_k(q)$ where q is a quadratic form over a field k of characteristic different from 2, L is a finite extension of k and $G_k(q)$ is the set of similarity factors of q in k . If $D_k(q)$ denotes the set of values of q in k , and $[D_k(q)]$ is the subgroup it generates in k , another classical result, this time due to Knebusch, gives $N_{L/k}(D_L(q_L)) \subset [D_k(q_k)]$. In a 1993 paper, Gille proved a norm principle which generalized that of Knebusch and also implied a partial version of Scharlau's norm principle. Then in 1996, Merkurjev proved a norm principle which implied Gille's norm principle. Gille considered a k -isogeny of semisimple algebraic groups $1 \rightarrow \mu \rightarrow G' \rightarrow G \rightarrow 1$ over a field k of characteristic 0. He showed that the norm of the image of $RG(L)$ in $H^1(L, \mu)$ under the usual connecting map, is in the image of $RG(k)$ in $H^1(k, \mu)$ where $RG(k)$ is the set of elements of $G(k)$ R -equivalent to 1. Merkurjev considered short exact sequences $1 \rightarrow G_1 \rightarrow G \rightarrow T \rightarrow 1$ where G_1 and G are connected reductive groups and T is an algebraic torus defined over a perfect field k . He showed that the norm of the image of $RG(L)$ in $T(L)$ is in the image of $RG(k)$ in $T(k)$. We will discuss Gille and Merkurjev's norm principles and show how they imply at least partial versions of the previous results. Then we will illustrate how Merkurjev used his norm principle to give an explicit description of the image of $RG(k)$ in $T(k)$ and give that description in some specific examples.

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