Dissertation Defense

Assimilation of velocity data into fluid dynamics simulations, an application to computational hemodynamics

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Abstract: Cardiovascular applications recently gave a strong impulse to numerical methods for fluid dynamics. Furthermore, thanks to new precise measurement devices and efficient image processing techniques, medicine is experiencing a tremendous increment of available data, inevitably affected by noise. Beyond validation, these data can be combined with numerical simulations in order to develop mathematical tools, known as data assimilation (DA) methods, of clinical impact. In the context of hemodynamics accuracy and reliability of assimilated solutions are particularly crucial in view of possible applications in the clinical routine. Hence, it is of central relevance to quantify the uncertainty of numerical results.

We propose a robust DA technique for the inclusion of noisy velocity measures, collected from magnetic resonance imaging, into the simulation of hemodynamics equations, namely the incompressible Navier-Stokes equations (NSE). The technique is formulated as a control problem where a weighted misfit between velocity and data is minimized under the constraint of the NSE; the optimization problem is solved with a discretize then optimize approach relying on the finite element method. The control variable is the normal stress on the inflow section of the vessel, which is usually unknown in real applications. We design deterministic and statistical estimators (the latter based on a Bayesian approach to inverse problems) for the estimation of the blood velocity and its statistical properties and of related variables of medical relevance, such as the wall shear stress. We also derive conditions on data location that guarantee the existence of an optimal solution.

Numerical simulations on 2-dimensional and axisymmetric 3-dimensional geometries show the consistency and accuracy of the method with synthetic noise-free and noisy data. Simulations on 2-dimensional geometries approximating blood vessels demonstrate the applicability of the approach for hemodynamics applications.

Friday, November 4, 2011, 4:00 pm
Mathematics and Science Center: W301

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