Colloquium

Brauer-Manin obstruction and integral points

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Abstract: In 1970, Yuri Manin showed how to combine class field theory and the Brauer-Grothendieck group to understand the structure of many counterexamples to the local-global principle for rational points of varieties defined over a number field. Since then, quite a few developments have taken place, including the study of weak approximation, descent, and torsors.
In 2005 one started using such methods to study existence and density of integral points of affine varieties. This is what I shall report on.
For principal homogeneous spaces under linear algebraic groups, when the groups are simply connected, we have the local-global principle, and, under some assumption of non-compactness, we have an extension of the Chinese remainder theorem, namely strong approximation for integral points (Kneser). This has now been combined with the Brauer-Manin approach and class field theory, leading to some control of integral points (existence and density in the completions) over nearly arbitrary homogeneous spaces of linear algebraic groups (F. Xu and the speaker, D. Harari, M. Bororovoi, C. Demarche).
Beyond the world of homogeneous spaces, there are conjectures for affine curves (Harari and Voloch) and computations for affine cubic surfaces (Wittenberg and the speaker). I shall thus comment on the question: which integers are sums of three cubes of integers?
I shall end the talk with a report on recent results on the integral points on some affine varieties which admit a pencil of homogeneous spaces, but are not homogeneous spaces themselves. A concrete example is given by equations $P(t) = q(x, y, z)$ with $P(t)$ an integral polynomial in one variable and $q(x, y, z)$ an indefinite ternary quadratic form (F. Xu and the speaker).

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