# Dissertation Defense
# Seminar

## *Problems on Sidon sets of integers*

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**Abstract:** A set $A$ of non-negative integers is a *Sidon set* if all the sums $a_1 + a_2$, with $a_1 \leq a_2$ and $a_1$, $a_2 \in A$, are distinct. In this dissertation, we deal with three results on Sidon sets: two results are about finite Sidon sets in $[n] = \{0, 1, \cdots, n-1\}$ and the last one is about infinite Sidon sets in $\mathbb{N}$ (the set of natural numbers).

First, we consider the problem of Cameron–Erdős estimating the number of Sidon sets in $[n]$. We obtain an upper bound $2^{c\sqrt{n}}$ on the number of Sidon sets which is sharp with the previous lower bound up to a constant factor in the exponent.

Next, we study the maximum size of Sidon sets contained in sparse random sets $R \subset [n]$. Let $R = [n]_m$ be a uniformly chosen, random $m$-element subset of $[n]$. Let $F([n]_m) = \max\{|S| \colon S \subset [n]_m \text{ is Sidon}\}$. Fix a constant $0 \leq a \leq 1$ and suppose $m = (1 + o(1))n^a$. We show that there is a constant $b = b(a)$ for which $F([n]_m) = n^{b+o(1)}$ almost surely and we determine $b = b(a)$. Surprisingly, between two points $a = 1/3$ and $a = 2/3$, the function $b = b(a)$ is constant.

Next, we deal with infinite Sidon sets in sparse random subsets of $\mathbb{N}$. Fix $0 < \delta \leq 1$, and let $R = R_\delta$ be the set obtained by choosing each element $i \subset \mathbb{N}$ independently with probability $i^{-1+\delta}$. We show that for every $0 < \delta \leq 2/3$ there exists a constant $c = c(\delta)$ such that a random set $R$ satisfies the following with probability 1:

- Every Sidon set $S \subset R$ satisfies that $|S \cap [n]| \leq n^{c+o(1)}$ for every sufficiently large $n$.
- There exists a large Sidon set $S \subset R$ such that $|S \cap [n]| \geq n^{c+o(1)}$ for every sufficiently large $n$.

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Mathematics and Science Center: W304

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