

DISSERTATION  
DEFENSE

*Scalable Efficient Methods for Incompressible Fluid-dynamics  
in Engineering Problems*

Umberto Villa  
Emory University

**Abstract:** Accurate and effective methods for the numerical solution of incompressible fluid dynamics is an old but still important challenging problem, as more and more complex problems in engineering biology, ecology, medicine, sport are tackled with computational methods.

In this dissertation defense, we investigate efficient solvers for two important models that governs the motion of a fluid, the incompressible Navier-Stokes and the Brinkman equations. The former describes the motion of an incompressible fluid in either an open or closed domain. The latter is used for describing the dynamics in a matrix of an inhomogeneous porous media, alternating bubbles and open channels.

For the solution of the unsteady Navier-Stokes Equations, we move from the pressure correction algebraic factorization formerly proposed by Saleri, Veneziani (2005). These schemes feature an intrinsic hierarchical nature, such that an accurate approximation of the pressure Schur complement is obtained by computing intermediate low-order guesses. The difference between the pressure at two successive correction steps provides a natural a-posteriori estimator for time adaptivity with no additional computational cost.

For the solution of the Brinkman Equations, we follow the approach presented in Mardal, Winther (2011) to precondition symmetric saddle point problems in a Hilbert settings. More specifically, we first present a novel mixed formulation of the Brinkman problem, with improved stability properties, in which we introduce the flow's vorticity as additional unknown. Based on stability analysis of the problem in the  $H(\text{curl}) - H(\text{div}) - L2$  norms, we derive a scalable block diagonal preconditioner which is optimal in the constant coefficient case.

Algorithms and preconditioners analysed in this thesis have been implemented in a parallel C++ code, using the finite element libraries LifeV and MFEM, and the linear algebra libraries Trilinos and HYPRE. We emphasize the performance of the proposed algorithms in solving problems of practical interest, involving complex geometries and realistic flow conditions. Numerical experiments in 2D and 3D confirm the effectiveness of our approach showing good efficiency and parallel scalability properties of the solvers proposed.

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Advisor: Alessandro Veneziani

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