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*The distribution of 2-Selmer ranks and additive functions*

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**Abstract:** The problem of determining the distribution of the 2-Selmer ranks of quadratic twists of an elliptic curve has received a great deal of recent attention, both in works conjecturing distributions and in those providing solutions; in both cases, the nature of the two-torsion of the elliptic curve plays a crucial role. In particular, if  $E/\mathbb{Q}$  has full two-torsion, the distribution is known, due to work of Heath-Brown, Swinnerton-Dyer, and Kane, and if  $E$  possesses no two-torsion, then, again, the distribution is known, due to work of Klagsbrun, Mazur, and Rubin, though with the caveat that one arranges discriminants in a non-standard way. In stark contrast to these two cases, we show that if  $K$  is a number field and  $E/K$  is an elliptic curve with partial two-torsion, then no limiting distribution on 2-Selmer ranks exists. We do so by showing that, for any fixed integer  $r$ , at least half of the twists of  $E$  have 2-Selmer rank greater than  $r$ , and we establish an analogous result for simultaneous twists, either for multiple elliptic curves twisted by the same discriminant or for a single elliptic curve twisted by a tuple of discriminants. These results depend upon connecting the 2-Selmer rank of twists to the values of an additive function and then establishing results analogous to the classical Erdős-Kac theorem. This work is joint with Zev Klagsbrun.

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