Combinatorics Seminar

On Erdos' conjecture on the number of edges in 5-cycles

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Abstract: Erdos, Faudree, and Rousseau (1992) showed that a graph on n vertices and at least $[n^2/4] + 1$ edges has at least $2[n/2] + 1$ edges on triangles and this result is sharp. They also considered a conjecture of Erdos that such a graph can have at most $n^2/36$ non-pentagonal edges with an extremal graph having two components, a complete graph on $[2n/3] + 1$ vertices and a complete bipartite graph on the rest. This was mentioned in other papers of Erdos and also it is No. 27 in Fan Chung's problem book.

In this talk we give a graph of $[n^2/4] + 1$ edges with much more, namely $n^2/8(2 + \sqrt{2}) + O(n) = n^2/27.31$ pentagonal edges, disproving the original conjecture. We show that this coefficient is asymptotically the best possible.

Given graphs H and F let $E_0(H, F)$ denote the set of edges of H which do not appear in a subgraph isomorphic to F, and let $h(n, e, F)$ denote the maximum of $|E_0(H, F)|$ among all graphs H of n vertices and e edges. We asymptotically determine $h(n, cn^2, C_3)$ and $h(n, cn^2, C_5)$ for fixed c, $1/4 < c < 1/2$. For $2k + 1 \geq 7$ we establish the conjecture of Erdos et al. that $h(n, cn^2, C_{2k+1})$ is obtained from the above two-component example.

One of our main tools (beside Szemeredi's regularity) is a new version of Zykov's symmetrization what we can apply for more graphs, simultaneously.

Friday, December 6, 2013, 4:00 pm
Mathematics and Science Center: W306

This is joint work with Zeinab Maleki.

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