Dissertation Defense

An epsilon improvement to the asymptotic density of k-critical graphs

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Abstract: Given a graph G the *chromatic number*, denoted $\chi(G)$, is smallest number of colors necessary to color $V(G)$ such that no adjacent vertices receive the same color. A graph G is k-critical if $\chi(G) = k$ but every proper subgraph has chromatic number less than k. As k-critical graphs can be viewed as minimal examples of graphs with chromatic number k, it is natural to ask how small such a graph can be. Let $f_k(n)$ denote the minimum number of edges in a k-critical graph on n vertices. The Ore construction, used to build larger k-critical graphs, implies that

$$f_k(n+k-1) \leq f_k(n) + (k-1)\left(\frac{k}{2} - \frac{1}{k-1}\right).$$

A recent paper by Kostochka and Yancey provides a lower bound for $f_k(n)$ which implies that the asymptotic density $\phi_k := \lim_{n\to\infty} f_k(n)/n = \frac{k}{2} - \frac{1}{k-1}$. In this work, we use the method of discharging to prove a lower bound on the number of edges which includes structural information about the graph. This lower bound shows that the asymptotic density of a k-critical graph can be increased by $\epsilon > 0$ by restricting to (K_{k-2})-free k-critical graphs.

We also prove that the graphs constructible from the Ore construction and K_k, called k-*Ore* graphs, are precisely the graphs which attain Kostochka and Yancey's bound. Moreover, we also provide results regarding subgraphs which must exist in k-Ore graphs. For the discharging argument, carried out in two stages, we also prove results regarding the density of nearly-bipartite subgraphs in k-critical graphs.

Wednesday, April 8, 2015, 4:30 pm
Mathematics and Science Center: W303

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