Algebra and Number Theory Seminar

A generalization of the Euler-Glaisher bijection

Andrew Sills
Georgia Southern University

Abstract: In 1748, Euler published his Introductio in Analysin Infinitorum. Chapter 16 of this work is the first systematic study of integer partitions in the mathematical literature. In it, he introduces infinite product generating functions and uses them to derive what is now known as Eulers partition identity, an English translation of which reads as follows: The number of different ways a given number can be expressed as the sum of different whole numbers is the same as the number of ways in which the same number can be expressed as the sum of odd numbers, whether the same of different. In modern terminology, the preceding is rephrased as the number of partitions of n into distinct parts equals the number of partitions of n into odd parts.
In 1883, J.W.L. Glaisher published the first bijective proof of Eulers partition identity, along with a natural generalization: the number of partitions of n where no part appears more than m - 1 times equals the number of partitions of n where no part is divisible by m. By combining a construction of P.A. MacMahon called partitions of infinity and knowledge of George Andrews' partition ideals of order 1 with Glaishers bijective proof of Eulers identity, we are led to discover a large class of partition identities with straightforward bijective proofs. This is joint work with James Sellers and Gary Mullen of Penn State. All terms will be defined and illustrated with concrete examples, so the required mathematical background will be minimal, and the talk should be accessible to all graduate students.

Tuesday, November 3, 2015, 4:00 pm
Mathematics and Science Center: W304

Mathematics and Computer Science
Emory University