

DISSERTATION
DEFENSE

Brauer–Manin Computations for Surfaces

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Abstract: The nature of rational solutions to polynomial equations is one which is fundamental to Number Theory and more generally, to Mathematics. Given the straightforward nature of this problem, one may be surprised by the difficulty when it comes to producing solutions.

The Hasse principle states that if an equation has local solutions everywhere then there is a global solution. Polynomials rarely satisfy this property. However Colliot-Thélène conjectures that another test on local solutions, the Brauer–Manin obstruction, exists for every rationally connected, smooth, projective, geometrically integral variety failing to satisfy the Hasse Principle.

We wish to explore the existence of a Brauer–Manin obstruction to the Hasse principle for certain families of surfaces. The first of which is a cubic surface written down by Birch and Swinnerton-Dyer in 1975,

$$\text{Norm}_{L/K}(ax + by + \phi z + \psi w) = (cx + dy)\text{Norm}_{K/k}(x + \theta y).$$

The left-hand side of this equality is a cubic norm and the right-hand side contains a quadratic norm. They make a correspondence between this failure and the Brauer–Manin obstruction, recently discovered by Manin, in a few specific instances. Using techniques developed in the ensuing 40 years, we show that a much wider class of norm form cubic surfaces have a Brauer–Manin obstruction to the Hasse principle, thus verifying the Colliot-Thélène conjecture for infinitely many cubic surfaces. The second family is a general set of diagonal K3 surfaces,

$$w^2 = ax^6 + by^6 + cz^6 + dw^6,$$

defined as varieties in weighted projective space. This section focuses on the particular geometry of these surfaces, verifying that their Picard rank is generically 19. We conclude by computing the Galois cohomology group, $H^1(\text{Gal}(\bar{k}/k), \text{Pic}\bar{X}) \simeq (\mathbf{Z}/2\mathbf{Z})^3$. The computation of this group is fundamental to determining the existence of a Brauer–Manin obstruction.

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