Dissertation Defense

Some Ramsey-type Theorems

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Abstract: Extending the concept of the Ramsey numbers, Erdős and Rogers introduced the function

$$f_{s,t}(n) = \min\{\max\{|W| : W \subseteq V(G) \text{ and } G[W] \text{ contains no } K_s\}\},$$

where the minimum is taken over all K_t-free graphs G of order n. We establish that for every $s \geq 3$ there exist constants c_1 and c_2 such that $f_{s,s+1}(n) \leq c_1(\log n)^{c_2}\sqrt{n}$. We also prove that for all $t-2 \geq s \geq 4$, there exists a constant c_3 such that $f_{s,t}(n) \leq c_3\sqrt{n}$. In doing so, we give a partial answer to a question of Erdős.

Another question of Erdős, answered by Rödl and Ruciński, asks if for every pair of positive integers ℓ and k, there exist a graph H having girth k and the property that every ℓ-coloring of the edges of H yields a monochromatic cycle C_k. Here, we establish that such a graph exists with at most $r^{O(k^2)}k^{O(k^3)}$ vertices, where $r = r_\ell(C_k)$ is the ℓ color Ramsey number for the cycle C_k. We also consider two closely related problems.

Finally, for a graph S, the h-subdivision $S^{(h)}$ is obtained by replacing each edge with a path of length $h+1$. For any graph S of maximum degree d on $s \geq s_0(h,d,\ell)$ vertices, we show there exists a graph G with $(\log s)^{20h}s^{1+1/(h+1)}$ edges having the following *Ramsey* property: any coloring of the edges of G with ℓ colors yields a monochromatic copy of the subdivided graph $S^{(h)}$. This result complements work of Pak regarding 'long' subdivisions of bounded degree.

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