Dissertation Defense

Elliptic curves, eta-quotients and Weierstrass mock modular forms

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Abstract: The relationship between elliptic curves and modular forms informs many modern mathematical discussions, including the solution of Fermat's Last Theorem and the Birch and Swinnerton-Dyer Conjecture. In this thesis we explore properties of elliptic curves, a particular family of modular forms called eta-quotients and the relationships between them. We begin by discussing elliptic curves, specifically considering the question of which quadratic fields have elliptic curves with everywhere good reduction. By revisiting work of Setzer, we expand on congruence conditions that determine the real and imaginary quadratic fields with elliptic curves of everywhere good reduction and rational j-invariant. Using this, we determine the density of such real and imaginary fields. In the next chapter, we begin investigating the properties of eta-quotients and use this theory to prove a conjecture of Han related to the vanishing of coefficients of certain combinatorial functions. We prove the original conjecture that relates the vanishing of the hook lengths of partitions and the number of 3-core partitions to the coefficients of a third series by proving a general theorem about this phenomenon. Lastly, we will see how these eta-quotients relate to the Weierstrass mock modular forms associated with certain elliptic curves. Alfes, Griffin, Ono, and Rolen have shown that the harmonic Maass forms arising from Weierstrass ζ-functions associated to modular elliptic curves "encode" the vanishing and nonvanishing for central values and derivatives of twisted Hasse-Weil L-functions for elliptic curves. We construct a canonical harmonic Maass form for the five curves proven by Martin and Ono to have weight 2 newforms with complex multiplication that are eta-quotients. The holomorphic part of this harmonic Maass form is referred to as the Weierstrass mock modular form. We prove that the derivative of the Weierstrass mock modular form for these five curves is itself an eta-quotient or a twist of one.

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