Algebra
Seminar

The Distribution Of The Number Of Prime Factors With Restrictions - Variations Of The Classical Theme

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Abstract: The study of $\nu(n)$ the number of prime factors of n began with Hardy and Ramanujan in 1917 who showed that $\nu(n)$ has normal order $log\, log\, n$ regardless of whether the prime factors are counted singly or with multiplicity. Their ingenious proof of this utilized uniform upper bounds for $N_k(x)$, the number of integers up to x with $\nu(n) = k$. Two major results followed a few decades later - the Erdös-Kac theorem on the distribution more generally of additive functions, and the Sathe-Selberg theorems on the asymptotic behavior of $N_k(x)$ as k varies with x - a significant improvement of Landau's asymptotic estimate for $N_k(x)$ for fixed k. We shall consider the distribution of the number of prime factors by imposing certain restrictions - such as (i) requiring all prime factors of n to be $< y$ (the important case of smooth numbers), and (ii) considering only the prime factors $< y$, but for all integers. For (i), I showed in 1982 how an interesting variation of the classical theme with regard to the variance of $\nu(n)$ takes place when $\log\, x/log\, y$ is large, and this led to further work by Hildebrand, Tenenbaum, Hensley and myself on the Erdös-Kac Theorem for smooth numbers. Very recently, I noticed a surprising variation of the classical theme in the case (ii) with regard to the *local distribution*. Details of the asymptotic analysis of the local distribution in (ii) with emphasis on uniformity in y has been carried out (Fall 2016) by my PhD student Todd Molnar. Our approach involves the interplay of a variety of methods such as combinatorial counting, the Perron integral formula, Selberg's method, Buchstab iteration, and difference-differential equations to achieve uniformity. Tenenbaum has indicated recently in communication that by a careful analysis involving the Selberg-Delange method, the error terms can be improved in certain crucial ranges.

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