Honors Defense

A Study of Benford's Law for the Values of Arithmetic Functions

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Abstract: "Benford's Law characterizes the distribution of initial digits in large datasets across disciplines. Since its discovery by Simon Newcomb in 1881, Benford's Law has triggered tremendous studies. In this paper, we will start by introducing the history of Benford's Law and discussing in detail the explanations proposed by mathematicians on why various datasets are Benford. Such explanations include the Spread Hypothesis, the Geometric, the Scale-Invariance, and the Central Limit explanations. "To rigorously de ne Benford's Law and to motivate criteria for Benford sequences, we will provide fundamental theorems in uniform distribution modulo 1 in Chapter 2. We will state and prove criteria for checking uniform distribution, including Weyl's Criterion, Van der Corput's Di erence Theorem, as well as their corollaries.

"In Chapter 3, we will introduce the logarithm map, which allows us to reformulate Benford's Law with uniform distribution modulo 1 studied earlier. We will start by examining the case of base 10 only and then generalize to arbitrary bases. "Finally, we will elaborate on the idea of good functions. We will prove that good functions are Benford, which in turn enables us to nd a new class of Benford sequences. We will use this theorem to show that the partition function p(n) and the factorial sequence n! follow Benford's Law."

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