Combinatorics Seminar

Packing nearly optimal Ramsey $R(3,t)$ graphs

Lutz Warnke
Georgia Institute of Technology

Abstract: In a celebrated paper from 1995, Kim proved the Ramsey bound $R(3,t) \geq ct^2/\log t$ by constructing an n-vertex graph that is triangle-free and has independence number at most $C\sqrt{n \log n}$. We extend this result, which is best possible up to the value of the constants, by approximately decomposing the complete graph K_n into a packing of such nearly optimal Ramsey $R(3,t)$ graphs.

More precisely, for any $\epsilon > 0$ we find an edge-disjoint collection $(G_i)_i$ of n-vertex graphs $G_i \subseteq K_n$ such that (a) each G_i is triangle-free and has independence number at most $C_\epsilon\sqrt{n \log n}$, and (b) the union of all the G_i contains at least $(1-\epsilon)\binom{n}{2}$ edges. Our algorithmic proof proceeds by sequentially choosing the graphs G_i via a semi-random (Rödl nibble type) variation of the triangle-free process.

As an application we prove a conjecture of Fox, Grinshpun, Liebenau, Person and Szabó in Ramsey theory. In particular, denoting by $s_r(H)$ smallest minimum degree of r-Ramsey-minimal graphs for H, we close the logarithmic gap for $H = K_3$ and establish $s_r(K_3) = \Theta(r^2 \log r)$.

Joint work with He Guo.

Monday, October 23, 2017, 4:00 pm

Mathematics & Science Center – W302

Mathematics and Computer Science
Emory University