Algebra Seminar

Brauer classes supporting an involution

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Abstract: The construction of the Brauer group of a field can be generalized to (commutative) rings, and more generally to schemes, by replacing central simple algebras with Azumaya algebras. As in the case of fields, the Brauer group is an important cohomological invariant of the scheme, featuring, for instance, in the Manin obstruction for rational points.

Many of the properties of central simple algebras generalize to Azumaya algebras, but sometimes modifications are needed. For example, Albert characterized the central simple algebras admitting an involution of the first kind as those whose Brauer class is 2-torsion. While this fails for Azumaya algebras over a ring R, Saltman showed that the 2-torsion classes in the Brauer group of R are precisely those containing some representative admitting an involution of the first kind. Knus, Parimala and Srinivas later gave a quantitative version of this statement: If A is an Azumaya algebra of over R such that its Brauer class is 2-torsion, then there is an Azumaya algebra in the Brauer class of A that admits an involution and has degree 2*deg(A).

In this talk, we shall recall what are Azumaya algebras and how the Brauer group of a ring (or a scheme) is constructed. Then we will present a recent work with Asher Auel and Ben Williams where we use topological obstruction theory to show that the quantitative result of Knus, Parimala and Srinivas cannot be improved in general. Specifically, there are Azumaya algebras of degree 4 whose Brauer class is 2-torsion, but such that any algebra that is Brauer-equivalent to them and admits an involution has degree divisible by 8 = 2*4.

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