

ALGEBRA
SEMINAR

*(Lack of) Rank Growth of Elliptic Curves in Quartic
Extensions*

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Abstract: Let E be an elliptic curve over the rationals, and let $E(K)$ be the group of K -rational points of E over a number field K . The Mordell-Weil Theorem says that $E(K)$ factors as a finitely generated abelian group; the rank of the free abelian part, $rk(E/K)$, is the rank of E over K . We will consider the related notion of rank growth. That is, for an extension of number fields F/K , we will consider the quantity $rk(E/F) - rk(E/K)$. We will survey some results, conjectures, and hopes in this area and outline an approach to produce infinitely many S_4 and A_4 quartic extensions of the rationals for which an elliptic curve E does not gain rank. The approach adapts methods of Mazur, Rubin, and others to manipulate the 2-Selmer group of a thin family of quadratic twists of our starting E in a prescribed fashion. In doing so, we'll construct quadratic extensions for which we can understand the local conditions of the corresponding twist and such that the quadratic extensions parameterize the quartic extensions of interest.

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MATHEMATICS
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