Algebra and Number Theory Seminar

Strong u-invariant and Period-Index bound

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Abstract: For a central simple algebra A over K, there are two major invariants, viz., *period* and *index*.

For a field K, define the *Brauer l-dimension of K* for a prime number l, denoted by $\mathrm{Br}_l\mathrm{dim}(K)$, as the smallest $d \in \mathbb{N} \cup \{\infty\}$ such that for every finite field extension L/K and every central simple L-algebra A of period a power of l, we have that $\mathrm{ind}(A)$ divides $\mathrm{per}(A)^d$.

If K is a number field or a local field (a finite extension of the field of p-adic numbers $\mathbb{Q}_p$, for some prime number p), then classical results from class field theory tell us that $\mathrm{Br}_l\mathrm{dim}(K) = 1$. This invariant is expected to grow under a field extension, bounded by the transcendence degree. Some recent works in this area include that of Lieblich, Harbater-Hartmann-Krashen for K a complete discretely valued field, in the good characteristic case. In the bad characteristic case, for such fields K, Parimala-Suresh have given some bounds.

Also, the u-invariant of K, denoted by $u(K)$, is the maximal dimension of anisotropic quadratic forms over K. For example, $u(\mathbb{C}) = 1$; for F a non-real global or local field, we have $u(F) = 1, 2, 4,$ or 8, etc.

In this talk, I will present similar bounds for the $\mathrm{Br}_l\mathrm{dim}$ and the strong u-invariant of a complete non-Archimedean valued field K with residue field κ.

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