Combinatorics Seminar

Off-diagonal Ramsey numbers for slowly growing hypergraphs

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Abstract: For a k-uniform hypergraph F and a positive integer n, the Ramsey number $r(F,n)$ denotes the minimum N such that every N-vertex F-free k-uniform hypergraph contains an independent set of n vertices. A hypergraph is *slowly growing* if there is an ordering $e_1, e_2, \ldots, e_t$ of its edges such that $|e_i \setminus \bigcup_{j=1}^{i-1} e_j| \leq 1$ for each $i \in \{2, \ldots, t\}$. We prove that if $k \geq 3$ is fixed and F is any non k-partite slowly growing k-uniform hypergraph, then for $n \geq 2$,

$$r(F,n) = \Omega\Big(\frac{n^k}{(\log n)^{2k-2}}\Big).$$

In particular, we deduce that the off-diagonal Ramsey number $r(F_5, n)$ is of order $n^3/\text{polylog}(n)$, where F_5 is the triple system $\{123, 124, 345\}$. This is the only 3-uniform Berge triangle for which the polynomial power of its off-diagonal Ramsey number was not previously known. Our constructions use pseudorandom graphs, martingales, and hypergraph containers.

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