Analysis and PDE Seminar

Inverse Problem for Hyperbolic Partial Differential Operators on Riemannian Manifolds Without Boundary

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Abstract: In this talk we consider an inverse problem for a hyperbolic partial differential operator on a Riemannian manifold without boundary. Such a manifold can be either compact, like a sphere or torus, or unbounded, like Euclidean or Hyperbolic spaces The hyperbolic partial differential operator we are studying is a self-adjoint first order perturbation of the Riemannian wave operator. In particular, this operator has time-independent lower order terms which can be written in the form of a vector field and a function which models the magnetic and electric potentials respectively. Our goal is to recover the speed of sound as well as these lower order terms by sending lots of waves from some open set of the manifold and measuring these waves on the same open set. This is called the local source-to-solution map. In this talk I will introduce the natural obstruction for recovering the aforementioned quantities from such measurements. This is the gauge of the problem. I will outline a proof that shows that modulo this gauge we can recover the wave speed, together with the magnetic and electric potentials from the local source-to-solution map. Our proof is based on a variation of the celebrated boundary control method (BC-method) which was developed by Belishev and Kurylev, and used to solve Gel'fand's inverse boundary spectral problem: "Can you recover a Riemannian manifold with boundary from the spectral data of its Dirichlet-Laplacian?" The BC-method reduces the PDE-based inverse problem to a geometric inverse problem of recovering a Riemannian manifold from a family of distance functions. We will also outline the proof of this problem.

This talk is based on my earlier work with Tapio Helin, Matti Lassas and Lauri Oksanen and on an ongoing project with my PhD student Andrew Shedlock.

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