Numerical Analysis and Scientific Computing Seminar

Sharp Detection of Low-Dimensional Structure in Bayesian Inverse Problems via (Dimensional) Logarithmic Sobolev Inequalities

Matthew Li. PhD
Massachusetts Institute of Technology

Abstract: Statistical inference in high-dimensions - whether through measuretransport, sampling, or other algorithms—often hinges on identifying and leveraging low-dimensional structures. We present an approach that represents high-dimensional target measures as low-dimensional updates of a dominating reference measure. While determining the optimal "dimension reduction" in this sense is computationally intractable, we explore how logarithmic Sobolev and Poincaré inequalities, along with their generalizations, enable us to derive approximations with certifiable error guarantees that are typically satisfactory in practice. In the latter part of the talk, we address why such tools from Markov semigroup theory are relevant to dimension reduction. We demonstrate that the optimal functional inequality for the Kullback-Leibler divergence is the dimensional logarithmic Sobolev inequality, which intrinsically captures the low-dimensional update structure we aim to exploit. Finally, we illustrate applications of these concepts to modern Bayesian inverse problems, particularly those involving GAN-based generative priors. Time-permitting, we may also discuss the connection of these ideas to emergent feature learning properties empirically observed in deep neural networks. Based on joint work with Olivier Zahm, Youssef Marzouk, Tiangang Cui, and Fengyi Li.

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Mathematics
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