

DISCRETE MATHEMATICS
SEMINAR

The Turán Density of k -Uniform Tight Cycles

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Abstract: For any uniformity r and residue k modulo r , we give an exact characterization of the r -uniform hypergraphs that homomorphically avoid tight cycles of length k modulo r , in terms of colorings of $(r-1)$ -tuples of vertices. This generalizes the result that a graph avoids all odd closed walks if and only if it is bipartite, as well as a result of Kam'ev, Letzter, and Pokrovskiy in uniformity 3. In fact, our characterization applies to a much larger class of families than those of the form $\mathcal{C}_k(r) = \{r\text{-uniform tight cycles of length } k \text{ modulo } r\}$.

We also outline a general strategy to prove that, if \mathcal{C} is a family of tight-cycle-like hypergraphs (including but not limited to the families $\mathcal{C}_k(r)$) for which the above characterization applies, then all sufficiently long cycles in \mathcal{C} will have the same Turán density. We demonstrate an application of this framework, proving that there exists an integer L_0 such that for every $L \geq L_0$ not divisible by 4, the tight cycle $\mathcal{C}^k(4)_L$ has Turán density $1/2$.

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