Combinatorics
Seminar

Higher rank antipodality

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Abstract: Motivated by general probability theory, we say that the set X in $\mathbb{R}^d$ is *antipodal of rank* k, if for any $k+1$ elements $q_1, \dots q_{k+1} \in X$, there is an affine map from $\mathrm{conv} X$ to the k-dimensional simplex Δ_k that maps $q_1, \dots q_{k+1}$ onto the $k+1$ vertices of Δ_k. For $k = 1$, it coincides with the well-studied notion of (pairwise) antipodality introduced by Klee.

We consider the following natural generalization of Klee's problem on antipodal sets: What is the maximum size of an antipodal set of rank k in $\mathbb{R}^d$? We present a geometric characterization of antipodal sets of rank k and adapting the argument of Danzer and Grünbaum originally developed for the $k = 1$ case, we prove an upper bound which is exponential in the dimension. We point out that this problem can be connected to a classical question in computer science on finding *perfect hashes*, and it provides a lower bound on the maximum size, which is also exponential in the dimension. Joint work with Zsombor Szilágyi and Mihály Weiner.

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