EMORY MATHEMATICS DIRECTED READING PROGRAM

Program description

The Emory Math Directed Reading Program (DRP) is a graduate student-run program aiming to pair undergraduate students with graduate student mentors to read and learn material that is not typically offered in a traditional course setting. Undergraduate students are expected to work mostly independently to read the text and attempt exercises, then meet regularly with their graduate student mentor to discuss the material.

Steering Committee

If you have any questions or concerns, or suggestions for future DRP topics, please reach out to Ylli Andoni at ylli.andoni@emory.edu, Guangqiu Liang at guangqiu.liang@emory.edu, or Mitchell Scott at mitchell.scott@emory.edu.

New Proposed Topics and Descriptions

Students specify a topic(s) of interest when applying to the program, in order to be matched with an appropriate graduate mentor. While the other document has a list of selected topics that have been done before, below are topics that are newly proposed by graduate students, and they are very eager to do a project on. If a topic sounds interesting, please see the next page for more detailed sample descriptions.

Algebra.

- Coding Theory and Algebraic Geometry

Analysis and Geometry.

- Random Matrix Theory

Applied and Computational Math.

- Deep Generative Modeling
- Uncertainty Quantification for Inverse Problems
- Optimization in Imaging

Discrete Math.

- Generating Functions

Number Theory.

- Mathematical Cryptography

Course name: Coding Theory and Algebraic Geometry
Text: *Codes and Curves*, by Judy Walker
Prerequisites: Abstract Algebra I (Math 421)

Description: Whenever data is transmitted across a channel, errors are likely to occur. It is the goal of coding theory to find efficient ways of encoding the data so that these errors can be detected, or even corrected. Normally this is done using group theory or discrete math; however, we plan to use different course of action – algebraic geometric codes. The goal of this course is to introduce you to some of the basics of coding theory, algebraic geometry, and algebraic geometric codes.

If you have no idea what codes are, we could think of the serial number of a dollar bill or the International Standard Book Number (ISBN), which can uniquely identify the dollar bill or book you have with some redundancies to make counterfeiting harder or correct if the ISBM is smudged, respectively.

Course name: Random Matrix Theory
Text: Chapter 1-7 of *A First Course in Random Matrix Theory*, by Marc Potters and Jean-Philippe Bouchaud
Prerequisites: Undergraduate Probability (Math 361)

Description: Eigenvalues and eigenvectors tell the full story behind matrices across all areas of mathematics, especially if that matrix has structure. But what story can we get from matrices that consist of every element being randomly selected from a probability distribution? Do the eigenvalues even behave, or settle into a distribution? The answer is yes! In this course, we plan on looking at the structure of two major types of random matrices. We will see how to construct them, how their eigenvalues behave, and what we can say about the numerics and analysis of such spectral probabilities.

While this is a fun pure math topic combining all types of math, there are also practical applications as well. What if real world data, such as stock prices were modeled as random correlations between another stock? We will take the pure theory and apply it to mathematical finance. After all, the two authors run a hedge fund together, so clearly this is useful!

Course name: Deep Generative Modeling
Text: *Deep Generative Modeling*, by Jakub M. Tomczak
Prerequisites: Undergraduate Probability (Math 361)

Description: I just saw is a photo of George Washington riding a dinosaur across the surface of Mars, but how did they get that? Photography wasn't invented yet, dinosaurs and George Washington didn't live during the same time, and Mars is far away. This is an extreme example of generative modeling, where we assume that data has an underlying distribution. Combining supervised learning and unsupervised learning, the resulting paradigm is called deep generative modeling, which utilizes the generative perspective on perceiving the surrounding world. It assumes that each phenomenon is driven by an underlying generative process that defines a joint distribution over random variables and their stochastic interactions, i.e., how events occur and in what order. The ultimate aim of the course is to outline the most important techniques in deep generative modeling and, eventually, enable readers to formulate new models and implement them.

Course name: Uncertainty Quantification for Inverse Problems
Text: *Discrete Inverse Problems: Insight and Algorithms*, by Per Christian Hansen
Prerequisites: Numerical Analysis (Math 315)

Description: A forward problem would be given a set of all these parameters, find out the final state of the model. This seems rather straight forward as you can perform a simple matrix multiplication or function evaluation depending on the system. Conversely, an inverse problem is given the final output of the system, can you determine the parameters that caused this system to evolve like this. It sounds much harder because it is. An added issue with inverse problems is since the true parameters are unknown, how do you figure out how close your guess is to the true solution? This is where uncertainty quantification comes in. It allows you to measure how far you are away from the unknown solution using the techniques of inverse problems.

Course name: Optimization in Imaging
Text: *Deblurring Images: Matrices, Spectra, and Filtering*, by Per Christian Hansen, James G. Nagy, and Dianne P. O'Leary
Prerequisites: Numerical Analysis (Math 315)

Description: Do you ever try to take a picture on the go and you look at the camera later to find out the picture is blurry? What do you do in that case, as you cannot go back in time to get the picture again. One might naively try figuring out what blur pattern it is and undo it. In image processing, this would be inverting a blur matrix; however, due to rounding errors this leads to a garbage result. This is where filtering out noise and using techniques to recover the original picture comes in. These methods often involve some parameters that aren't known *a priori*, so this is where optimization comes in so that one can figure out the best parameters to unblur the matrix.

Course name: Generating Functions
Text: *generatingfunctionology*, by Herbert Wilf
Prerequisites: Introduction to Combinatorics (Math 330) or Complex Variables (Math 318)

Description: 0,1,1,2,3,5,8,13,.... A keen observer of mathematics might recognize this as the Fibonacci sequence. But what is F_n, the n^{th} number in the Fibonacci sequence? One could just recursively add numbers, but there is actually a function that can tell you the answer. Generating functions are a bridge between discrete mathematics, on the one hand, and continuous analysis (particularly complex variable theory) on the other. It is possible to study them solely as tools for solving discrete problems. As such there is much that is powerful and magical in the way generating functions give unified methods for handling such problems. The full beauty of the subject of generating functions emerges only from tuning in on both channels: the discrete and the continuous.

But how good is the generating function? Is it accurate? What can we do with this as an "answer"? Is it even an "answer"? In this course, we hope to convince you that answers like this one are often spectacularly good, in that they are themselves elegant, they allow you to do almost anything you'd like to do with your sequence, and generating functions can be simple and easy to handle even in cases where exact formulas might be stupendously complicated.

Course name: Mathematical Cryptography
Text: *An Introduction to Mathematical Cryptography*, by Joe Silverman, Jill Pipher, and Jeffery Hoffstein
Prerequisites: Abstract Algebra I (Math 421)

Description: Do you ever worry how safe your social security information is when you type it in an online application? No, you just do because it will be encrypted and useless if someone were to get ahold of it. This is because we put great trust in codes. Codes are all around us and have been since antiquity. Starting from the simplest Caesar sipher codes, to the state-of-the-art RSA cryptosystem or Diffe-Hellman, we will learn about what mathematically makes a code, and why some are harder for ne'er-do-wells to decode. We we delve deep into how the codes we use every day to protect our personal information actual work.