

Polynomial-to-Exponential Transition in Hypergraph Ramsey Numbers

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Let $r_k(s, e; t)$ denote the smallest N such that any red/blue edge coloring of the complete k -uniform hypergraph on N vertices contains either e red edges among some s vertices, or a blue clique of size t . Erdős and Hajnal introduced the study of this Ramsey number in 1972 and conjectured that for fixed $s > k \geq 3$, there is a well defined value $h_k(s)$ such that $r_k(s, h_k(s) - 1; t)$ is polynomial in t , while $r_k(s, h_k(s); t)$ is exponential in a power of t . Erdős later offered 500 dollars for a proof.

Conlon, Fox, and Sudakov proved the conjecture for $k = 3$ and 3-adically special values of s , and Mubayi and Razborov proved it for $s > k \geq 4$. We prove the conjecture for $k = 3$ and all s , settling all remaining cases of the problem.

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