

Rainbow paths in regular graphs

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Graham conjectured that any d distinct nonzero elements of a prime-order cyclic group can be ordered so that all d partial sums are distinct. In the Cayley digraph generated by these elements with its natural edge coloring, this conjecture equivalently asserts the existence of a rainbow directed path with $d - 1$ edges. In a similar vein, Schrijver asked if every properly edge-colored d -regular (undirected) graph contains a rainbow path with $d - 1$ edges. We resolve both of these problems asymptotically as d grows: in Graham's setting, we produce an ordering with $d - o(d)$ distinct partial sums, and in Schrijver's setting, we produce a rainbow path with $d - o(d)$ edges. This is joint work with Matija Bucić, Alp Müyesser, Alexey Pokrovskiy, and Liana Yepremyan.