

Nearly tight bounds for MaxCut in hypergraphs

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An r -cut of a k -uniform hypergraph is a partition of its vertex set into r parts, and the size of the cut is the number of edges which have at least one vertex in each part. The study of the possible size of the largest r -cut in a k -uniform hypergraph was initiated by Erdős and Kleitman in 1968. For graphs, a celebrated result of Edwards states that every m -edge graph has a 2-cut of size $m/2 + \Omega(m^{1/2})$, which is sharp. In other words, there exists a cut which exceeds the expected size of a random cut by the order of $m^{1/2}$. Conlon, Fox, Kwan and Sudakov proved that any k -uniform hypergraph with m hyperedges has an r -cut whose size is $\Omega(m^{5/9})$ larger than the expected size of a random r -cut, provided that $k \geq 4$ or $r \geq 3$. They further conjectured that this can be improved to $\Omega(m^{2/3})$. Recently, Raty and Tomon improved the bound $m^{5/9}$ to $m^{3/5-o(1)}$ when $r \in \{k-1, k\}$. Using a novel approach, we prove the following approximate version of the Conlon–Fox–Kwan–Sudakov conjecture: for each $\varepsilon > 0$, there is some $k_0 = k_0(\varepsilon)$ such that for all $k > k_0$ and $2 \leq r \leq k$, in every k -uniform hypergraph with m edges there exists an r -cut exceeding the random one by $\Omega(m^{2/3-\varepsilon})$. Moreover, we show that (if $k \geq 4$ or $r \geq 3$) every k -uniform linear hypergraph has an r -cut exceeding the random one by $\Omega(m^{3/4})$, which is tight and proves a conjecture of Raty and Tomon.

Joint work with Julien Portier.