Folkman graphs and finite geometry

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15 November 2025

Folkman's theorem asserts the existence of graphs G which are K_{s+1}-free, but which have the property that every two-coloring of $E(G)$ contains a monochromatic copy of K_s. The quantitative aspects of $f(s)$, the least n such that there exists an n-vertex graph with both properties above, are notoriously difficult; a series of improvements to $f(3)$, the smallest nontrivial Folkman number, over the span of two decades witnessed the solution to two \$100 Erdős problems, and the current record due to Lange, Radziszowski, and Xu now stands at $f(3) \leq 786$, the proof of which is computer-assisted. More generally, the bound $f(s) \leq c^{s^3}$ of Balogh and Samotij (proved using random graphs and hypergraph containers), where $c > 0$ is an absolute constant, is the state of the art for larger cliques.

In this talk, I will discuss Folkman-like properties of some pseudorandom graphs coming from finite geometry. These constructions present new potential paths to two fundamental questions in the area of restricted Ramsey theory: first, yet another \$100 problem of Ron Graham to show that $f(3) \leq 100$; and secondly, a longstanding folklore conjecture that $f(s) \leq C^s$ for some fixed $C > 0$. This is based on joint work with Steven van Overberghe and with Patrick Morris, Maya Sankar, and Liana Yepremyan.