Extremal Hypergraphs without Generalized 4-Cycles

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We study a problem posed by Erdős on determining the maximum number of edges in an r-uniform hypergraph where the unions of any two pairs of disjoint edges are distinct. The forbidden configuration, two pairs of disjoint edges with the same union, is known as a generalized 4-cycle. For $r \geq 4$ and sufficiently large n, Füredi conjectured that this maximum equals $\binom{n-1}{r-1} + \left\lfloor \frac{n-1}{r} \right\rfloor$. In this talk, we confirm this conjecture, characterize all the extremal hypergraphs, and discuss several related bipartite Turán problems in hypergraphs.