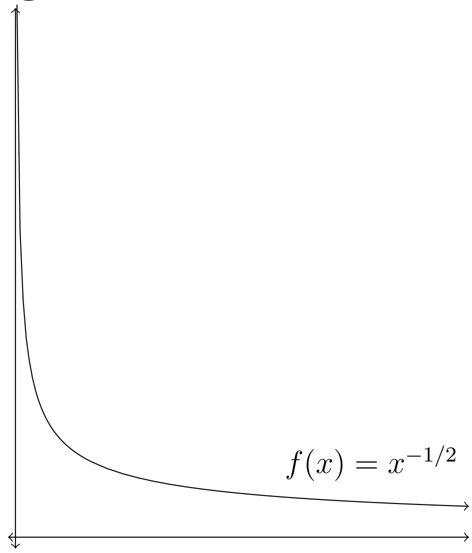
Groupwork 02/04: MATH 112 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed.

1 In Class Problems: Type II Improper Integrals

To the right is the plot of $f(x) = \dfrac{1}{x^{1/2}}$.



1.1 What is $\displaystyle\int_{1/2}^{1} \frac{1}{x^{1/2}}\, dx$?

1.2 What is $\displaystyle\int_{1/4}^{1} \frac{1}{x^{1/2}}\, dx$?

1.3 What is $\displaystyle\lim_{t\to 0}\int_{t}^{1} \frac{1}{x^{1/2}}\, dx$?

Definition: If $f(x)$ is continuous on $(a, b]$ and is discontinuous at a then $\displaystyle\int_a^b f(x)\, dx = \lim_{t\to a^+}\int_t^b f(x)\, dx$.

Definition: If $f(x)$ is continuous on $[a, b)$ and is discontinuous at bb then $\displaystyle\int_a^b f(x)\, dx = \lim_{t\to b^-}\int_a^t f(x)\, dx$.

Definition: If $f(x)$ is discontinuous at c, $a < c < b$ and $\displaystyle\int_a^c f(x)\, dx$ and $\displaystyle\int_c^b f(x)\, dx$ then we define $\displaystyle\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

1.4 Find $\displaystyle\int_0^1 \frac{1}{x^{1/2}}\, dx$

1.5 Find $\displaystyle\int_0^1 \frac{1}{x^{1}}\, dx$

1.6 Find $\displaystyle\int_0^1 \frac{1}{x^{2}}\, dx$

1.7 For what positive p does $\displaystyle\int_0^1 \frac{1}{x^{p}}\, dx$ converge?

1.8 Find $\displaystyle\int_0^1 \frac{e^{-1/x}}{x^{2}}\, dx$

1.9 Find $\displaystyle\int_0^1 \frac{\ln(x)}{x^{2}}\, dx$

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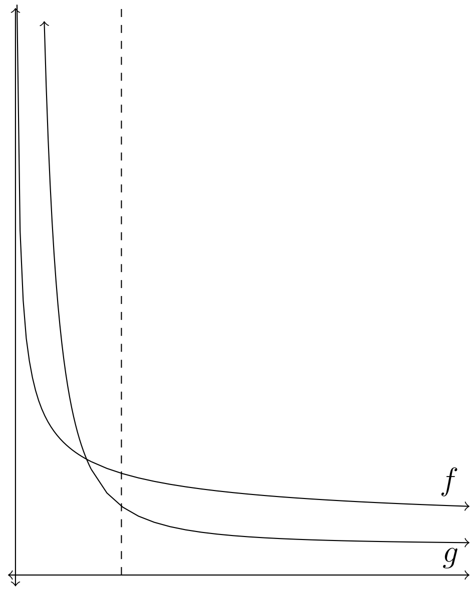
2 In Class Problems: The Comparison Test

To the right is the plot of two functions f and g



2.1 Find $\displaystyle\int_1^\infty \frac{1}{x^{1/2}}\,dx$

2.2 Find $\displaystyle\int_1^\infty \frac{1}{x^1}\,dx$

2.3 Find $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$

2.4 For what positive p does $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ converge?

Theorem (The Comparison Test): If $f(x) \geq g(x) \geq 0$ are continuous functions on $[a, \infty)$

a) If $\displaystyle\int_a^\infty f(x)\,dx$ converges then $\displaystyle\int_a^\infty g(x)\,dx$ converges

b) If $\displaystyle\int_a^\infty g(x)\,dx$ diverges then $\displaystyle\int_a^\infty f(x)\,dx$ diverges

2.5 Let $f(x) = \dfrac{1}{x^2}$ let $g(x) = \dfrac{x^2 - x + 1/4}{(x+1)(x+4)^2(x+7)}$. Use the comparison test to show that $\displaystyle\int_1^\infty g(x)\,dx$ converges. (Hint: show that the numerator of $g(x)$ is less than x^2 and the denominator is greater than x^4 for $x \geq 1$. Conclude that $g(x) < 1/x^2$ to get an answer)

2.6 Let $f(x) = \dfrac{x^5 + 2x^4 + x + 1}{(x^2 - x + 1)^2(x-4)(x-3)}$ and $g(x) = \dfrac{1}{x}$. Use the comparison test to show that $\displaystyle\int_{10}^\infty f(x)\,dx$ diverges. (Hint: show that the numerator of $f(x)$ is greater than x^5 and the denominator is less than x^6 for $x \geq 10$. Conclude that $f(x) > 1/x$ to get an answer)

2.7 (7.8.76) If $f(x)$ is continuous and $\displaystyle\int_{-\infty}^\infty f(x)\,dx$ is convergent where $\displaystyle\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^a f(x)\,dx + \int_a^\infty f(x)\,dx$ carefully show that for any $b < a$ or $b > a$ that

$$\int_{-\infty}^a f(x)\,dx + \int_a^\infty f(x)\,dx = \int_{-\infty}^b f(x)\,dx + \int_b^\infty f(x)\,dx$$

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