Groupwork 03/04: MATH 112 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed.

1 In Class Problems: Integral Test Remainder Estimates

1.1 What is $\displaystyle\int_2^\infty \frac{1}{x(ln(x))^2}\,dx$?

1.2 What is $\displaystyle\int_n^\infty \frac{1}{x(ln(x))^2}\,dx$?

1.3 Find the first n so $\displaystyle\int_n^\infty \frac{1}{x(ln(x))^2}\,dx < 0.001$?
(you may leave your answer as n is equal to e to some power)

Theorem:(Integral Test Remainder Estimate pg. 723) If $f(k) = a_k$, where f is positive, continuous and decreasing for $x \geq n$ and $\sum a_k$ is convergent (or a_k converges by the integral test with function f) then

$$\int_{n+1}^\infty f(x)\,dx \leq R_n \leq \int_n^\infty f(x)\,dx$$

where R_n is the remainder, or error, defined as

$$R_n = \sum_{k=n}^\infty a_k$$

An error estimate question will ask for what value of n must $R_n < r$ for some given r. To use the integral remainder estimate, first we need to show the function f satisfies the conditions for the integral test (positive, continuous and decreasing). Next, we find the first n where $\displaystyle\int_n^\infty f(x)\,dx < r$ then use the inequality $R_n \leq \displaystyle\int_n^\infty f(x)\,dx$ to find that for that n, it must be true that $R_n < r$.

1.4 How many terms of the series $\displaystyle\sum_{k=1}^\infty \frac{1}{n(\ln(n))^2}$ need to be added to have an error less than 0.01?

(you may leave your answer as n is equal to e to some power)

1.5 Using the same process find how many terms of the series $\displaystyle\sum_{k=1}^\infty \frac{1}{(n+1)^2}$ need to be added to have an error less than 0.002?

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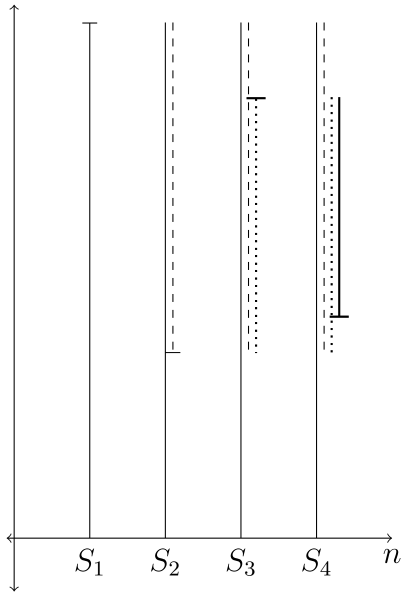
2 In Class Problems: Alternating Series Error Estimates

To the right is the plot of the partial sums S_n for the alternating series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\ln(k+1)}$



2.1 Show that $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\ln(k+1)}$ converges.

2.2 Find the n where $\frac{1}{\ln(n+1)} < 0.01$

(you may leave your answer as n is equal to e to some power)

Theorem: (Alternating Series Remainder Estimate pg. 735) If $\sum_{k=1}^{\infty}(-1)^{k+1}b_k$ is an alternating series where $\lim_{k\to\infty} b_k = 0$ and $b_k \geq b_{k+1}$ (or the series converges by the alternating series test), then

$$|R_n| \leq b_{n+1}$$

where R_n is the remainder, or error, defined as

$$R_n = \sum_{k=n}^{\infty}(-1)^{k+1}b_k$$

An error estimate question will ask for what value of n must $R_n < r$ for some given r. To use the alternating series remainder estimate we first show that $\lim_{n\to\infty} b_n = 0$ and $b_n \geq b_{n+1}$. Next, we find the first n where $b_n < r$ then use the inequality $|R_n| \leq b_{n+1}\, dx$ to find that for $n-1$, $R_n < r$.

2.3 How many terms of the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\ln(k+1)}$ need to be added to have an error less than 0.01?

(you may leave your answer as n is equal to e to some power)

2.4 How many terms of the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(k+1)^2}$ need to be added to have an error less than 0.0004?