Groupwork 04/08: MATH 112 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed.

1 In Class Problems: Taylor Series Remainder Estimates

Recall the definition of a Taylor series centered at a : $\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

1.1 Fill in the table for the derivatives of $f(x) = \ln(x)$.

n	$f^{(n)}(x)$
0	
1	
2	
3	
4	
5	
6	

1.2 Let $f(x) = \ln(x)$. Use the definition of Taylor series to write the first four terms of the Taylor series for $f(x)$ centered at $a = 1$. (This is called the fourth degree Taylor Polynomial)

Theorem:(Taylor Series Remainder Estimate pg. 762). Let $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)(a)}}{k!}(x-a)^k$ be a Taylor Series. Then for x in the interval $[a-d, a+d]$, the following is true:

$$|R_n(x)| \leq \max_{[a-d,a+d]} \frac{|f^{(n+1)}(x)|}{(n+1)!}|x-a|^{n+1}$$

where R_n is the remainder, or error, defined as

$$R_n(x) = \sum_{k=n+1}^{\infty} \frac{f^{(k)(a)}}{k!}(x-a)^k$$

for the Taylor polynomial

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)(a)}}{k!}(x-a)^k$$

An error estimate question will ask for the maximum error of the first n terms of a Taylor series in some range. We can think of the remainder estimate as finding the worst possible next term in that range. We can simplify this more. $(n+1)!$ does not change with x. The maximum of $|x-a|$ over $[a-d, a+d]$ is d. So we get the simplified Remainder estimate on the back page.

Theorem:(Taylor Series Remainder Estimate pg. 762). Let $f(x) = \sum_k = 0^\infty \frac{f^{(k)}(a)}{k!}(x-a)^k$ be a Taylor Series. Then for x in the interval $[a-d, a+d]$, the following is true:

$$|R_n(x)| \leq \frac{\max\limits_{[a-d,a+d]} |f^{(n+1)}(x)|}{(n+1)!}|d|^{n+1}$$

To find the remainder estimate all we need to find is the maximum of the absolute value of the $n+1$ derivative over $[a-d, a+d]$. Below we recap that process.

We want the maximum error of the fourth degree Talyor polynomial for $f(x) = \ln(x)$ over $[9/10, 11/10]$.

1.3 What is d?

1.4 We need to find $\max_{[a-d,a+d]} |f^{(5)}(x)|$ (since $n = 4$). The maximum of a function over a closed interval is either at a critical point or the endpoints of the interval.

1.4 a) To find the critical points of $f^5(x)$ we need to find where its derivative is zero or undefined and which, if any, are in the interval $[a-d, a+d]$. Where is $f^6(x)$ zero or undefined? Which, if any, are in the interval $[9/10, 11/10]$

1.4 b) Now find the values of $|f^{(5)}(x)|$ for $x = 9/10, x = 11/10$ and any critical points you find inside the interval $[9/10, 11/10]$. Which absolute value is the greatest?

1.5 Using your answer from 1.3 and 1.4 b), and using the formula

$$|R_n(x)| \leq \frac{\max\limits_{[a-d,a+d]} |f^{(n+1)}(x)|}{(n+1)!}|d|^{n+1}$$

find the maximum error of the fourth degree Talyor polynomial for $f(x) = \ln(x)$ over $[9/10, 11/10]$.

1.6 Find the fourth degree polynomial for $f(x) = \cos(2x)$ about $a = \pi/3$, and the maximum error over $[0, 2\pi/3]$.

1.7 Find the fourth degree polynomial for $f(x) = xe^(x)$ about $a = 1$, and the maximum error over $[0, 2]$.