Homework 1 1/16: MATH 112-1 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed. Collaboration with classmates is allowed, but all work submitted must be written out and explained by you.

3 Homework 1 Problems: Integration by Parts

3.1 Evaluate the following integrals.

3.1 a) (7.1.23) $\displaystyle\int_0^{1/2} x\cos(\pi x)\, dx$

3.1 b) (7.1.16) $\displaystyle\int \frac{z}{10^z}\, dz$

3.1 c) (7.1.8) $\displaystyle\int t^2\sin(\beta t)\, dt$

3.1 d) (7.1.35) $\displaystyle\int_1^2 x^4(\ln(x))^2\, dx$

3.1 e) (7.1.36) $\displaystyle\int_0^t e^s\sin(t-s)\, ds$

3.1 f) (7.1.19) $\displaystyle\int z^3e^z\, dz$

3.2 (7.1.51) Use integration by parts to prove the reduction formula:

$$\int (\ln(x))^n\, dx = x(\ln(x))^n - n\int (\ln(x))^{n-1}\, dx$$

3.3 (7.1.55) Use the reduction formula from 3.2 to find $\displaystyle\int (\ln(x))^3\, dx$

3.4 (7.1.69) A particle that moves along a straight line has velocity $v(t) = t^2e^{-t}$ meters per second after t seconds. How far will it travel during the first t seconds?

3.5 First make a substitution and then use integration by parts to evaluate the following integrals.

3.6 a) (7.1.37) $\displaystyle\int e^{\sqrt{x}}\, dx$

3.6 b) (7.1.39) $\displaystyle\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3\cos(\theta^2)\, d\theta$

Extra Problems 1/16: MATH 112-1 Prof. Maxwell Auerbach

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4 Extra Problems: Integration by Parts

4.1 Solve the following integrals using any applicable method.

4.1 a) (7.5.15) $\int t\sec(t)\tan(t)\,dt$

4.1 b) (7.5.28) $\int \sin(\sqrt{at})\,dt$

4.1 c) (7.5.3) $\int_1^4 \sqrt{y}\ln(y)\,dy$

4.1 d) (7.5.45) $\int x^5 e^{-x^3}\,dx$

4.1 e) (7.5.2) $\int_0^1 (3x+1)^{\sqrt{2}}\,dx$

4.1 f) (original) $\int \sin^3(x)\cos(x)\,dx$

4.2 (original) Our goal is to show that $\int_{-c}^{c} \sin(x)x^{2n}\,dx = 0$ for any positive integer n and number c.

4.2 a) Show that $\int_{-c}^{c} \sin(x)\,dx = 0$ for any number c. (Hint: recall that $\sin(x)$ is an odd function).

4.2 b) Let $f(x)$ be a smooth function such that $f(-a) = -f(a)$ and $f'(a) = f'(-a)$. Show that $\int_{-a}^{a} f''(x)x^{2n}\,dx = 2n\cdot(2n-1)\int_{-a}^{a} f(x)x^{2n-2}\,dx$.

4.2 c) Recall the notation that $k! = k\cdot(k-1)\cdot(k-2)\cdots 2\cdot 1$. Use the above to show that $\int_{-c}^{c} \sin(x)x^{2n}\,dx = (2n)!\int_{-c}^{c} \sin(x)\,dx = 0$ for any positive integer n.

4.3 (7.1.70) If $f(0) = 0 = g(0)$ and f'' and g'' are continuous, show that

$$\int_0^a f(x)g''(x)\,dx = f(a)g'(a) - f'(a)g(a) + \int_0^a f''(x)g(x)\,dx$$

4.4 (original) The industrious people of Wyoming, Ohio are teaching themselves to make screws. They find that the rate they make screws is described by the function $w(t) = (t^3 - 7t - 6)e^{2t}$ screws per weeks where t is time in weeks since they started making screws.

4.4 a) Wanjala wants to find the function that describes the total number of screws produced in their hometown of Wyoming Ohio. Describe what the mathematical process, including the techniques and theorems used, that could find Wanjala the function that describes the total number of screws produced in Wyoming, Ohio.

4.4 b) What is the function that describes the total number of screws produced in Wyoming, Ohio? (Hint: how may screws were produced before time $t = 0$)

4.5 (original) Camille finds that her car is accelerating according to the function $c(t) = (t+\pi/4)\sin(t+\pi/4)$ miles per hour squared. Assuming that Camille's car was traveling at 0 miles per hour at time $t = 0$ hours, how far did in miles did Camille's car travel during the first π hours.