Homework 3 1/30: MATH 112-1 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed. Collaboration with classmates is allowed, but all work submitted must be written out and explained by you.

2 Homework 3 Problems: Improper Integrals

2.1 Determine whether the following integral is convergent or divergent. Evaluate the convergent integrals.

2.1 a) (7.8.17) $\displaystyle\int_1^{\infty} \frac{1}{x^2+x}\,dx$

2.1 d) (7.8.25) $\displaystyle\int_0^{\infty} e^{-\sqrt{y}}\,dy$

2.1 b) (7.8.10) $\displaystyle\int_{-\infty}^{0} 2^r\,dr$

2.1 e) (7.8.22) $\displaystyle\int_1^{\infty} \frac{\ln(x)}{x^2}\,dx$

2.1 c) (7.8.18) $\displaystyle\int_2^{\infty} \frac{1}{v^2-2v+3}\,dv$

2.1 f) (7.8.26) $\displaystyle\int_1^{\infty} \frac{1}{\sqrt{x}+x\sqrt{x}}\,dx$

2.2 (7.8.71 modified) Determine how large the number a has to be so that (Hint: solve the integral on the left hand side. Set your answer into the inequality, then use the quadratic formula to find a solution. You do not need to simplify your answer besides setting $a =$ something)

$$\int_a^{\infty} \frac{2x^2+2x+5}{(x^2+4x+4)(x^2-2x+1)}\,dx < \frac{1}{3}$$

3 Homework 3 Problems: Partial Fraction Decomposition

3.1 Find the following integrals.

3.2 a) (7.4.18) $\displaystyle\int_1^{2} \frac{3x^2+6x+2}{x^2+3x+2}\,dx$

3.2 b) (7.4.15) $\displaystyle\int_{-1}^{0} \frac{x^3-4x+1}{x^2-3x+2}\,dx$

Extra Problems 1/30: MATH 112-1 Prof. Maxwell Auerbach

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4 Extra Problems: Cumulative Integration

4.1 Find the following integrals. If they are an improper integral state whether they are convergent or divergent, and if they converge find the value they converge to

4.1 a) (7.8.21) $\displaystyle\int_1^\infty \frac{\ln(x)}{x}\,dx$

4.1 b) (7.4.21 modified) $\displaystyle\int_2^\infty \frac{1}{(t^2-1)^2}\,dx$

4.1 c) (original) $\displaystyle\int \frac{x^4}{x^2-1}\,dx$

4.1 d) (original) $\displaystyle\int \frac{2-x}{x^4+x^2}\,dx$

4.1 e) (7.8.14) $\displaystyle\int_0^\infty \frac{e^{-1/x}}{x^2}\,dx$

4.1 f) (original) $\displaystyle\int_{-1}^1 \frac{3x^2-x+2}{(x-2)^2(x+2)}\,dx$

4.1 g) (7.5.66) $\displaystyle\int_{\pi/4}^{\pi/3} \frac{\ln(\tan(x))}{\sin(x)\cos(x)}\,dx$

4.1 h) (original) $\displaystyle\int_2^\infty e^{-x}\,dx$

4.1 i) (original) $\displaystyle\int_{-2}^0 \frac{x^5-x-1}{(x-1)^2}\,dx$

4.1 j) (original) $\displaystyle\int \frac{x^2+2}{(x+1)(x+4)^2}\,dx$

4.2 (7.8.62) The average speed of molecules in an ideal gas is

$$\overline{v} = \frac{4}{\pi}\left(\frac{M}{2RT}\right)^{3/2}\int_0^\infty v^3 e^{-Mv^2/(2RT)}\,dv$$

where M is the molecular weight of the gas, R is the gas constant, T is the gas temperature, and v is the molecular speed. Show that

$$\overline{v} = \sqrt{\frac{8RT}{\pi M}}$$

4.3 (original) In a study of the spread of illicit drug use from an enthusiastic user to a population of N users, the authors model the number of expected new users by the equation

$$\gamma = \int_0^\infty \frac{cN(1-e^{-kt})}{k}e^{-\lambda t}\,dt$$

where c, k and λ are positive constants. Evaluate this integral to express γ in terms of c, N, k, and λ.

4.4 (7.4.53 modified) Use integration by parts, alongside rational decomposition to solve the integral $\displaystyle\int \ln(x^3-x^2+x-1)\,dx$