Homework 5 2/11: MATH 112-1 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed. Collaboration with classmates is allowed, but all work submitted must be written out and explained by you.

1 Homework 5 Problems: Starting Sequences

1.1 List the first five terms of the sequence

1.1 a) (11.1.3)
$$a_n = \frac{2^n}{2n+1}$$

1.1 b) (11.1.11)
$$a_1 = 2,$$
$$a_{n+1} = \frac{a_n}{1+a_n}$$

1.1 c) (11.1.7)
$$a_n = \frac{1}{(n+1)!}$$

1.2 Find a formula for the general term a_n of the sequence, assuming the pattern of the first few terms continues.

1.2 a) (11.1.16) $\{5, 8, 11, 14, 17, \ldots\}$

1.2 b) (11.1.15) $\left\{4, -1, \frac{1}{4}, \frac{-1}{16}, \frac{1}{64} \ldots\right\}$

1.3 Determine whether the following sequence converges or diverges. If it converges, find the limit. Make sure to show all work, and cite any results from class used.

1.3 a) (11.1.24) $a_n = \dfrac{3+5n^2}{1+n}$

1.3 b) (11.1.35) $a_n = \dfrac{(-1)^n}{2\sqrt{n}}$

1.4 (original) Find what values of r is the sequence $\{r^n\}$ convergent? (hint: see page 700 of the book)

Extra Problems 2/11: MATH 112-1 Prof. Maxwell Auerbach

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2 Extra Problems: Starting Sequences

2.1 If \$1000 is invested at 6% interest, compounded annually, then after n years the investment is worth $a_n = 1000(1.06)^n$ dollars.

2.1 a) Find the first five terms in the sequence $\{a_n\}$

2.1 b) Is the sequence convergent or divergent? Explain.

2.2 Determine whether the following sequence converges or diverges. If it converges, find the limit. Make sure to show all work, and cite any results from class used.

2.2 a) (11.1.27) $a_n = 3^n 7^{-n}$

2.2 b) (original) $a_n = \left\{(-14)^n 4^{-2n}\right\}$

2.2 c) (11.1.26) $a_n = \dfrac{3\sqrt{n}}{\sqrt{n}+2}$

2.2 d) (original) $a_n = \left\{(-1)^n \ln(1+n^{-2})\right\}$

2.2 e) (11.1.46) $a_n = 2^{-n}\cos(\pi n)$

2.2 f) (11.1.49) $a_n = \ln(2n^2+1) - \ln(n^2+1)$

2.2 g) (original) $a_n = \left\{\ln(n^3+2)e^{-n/4}\right\}$

2.2 h) (11.1.40) $a_n = \dfrac{\arctan(n)}{n}$

2.3 (11.1.83) Fibonacci posed the following problem: Suppose that rabbits live forever and that every month each pair produces a new pair which becomes productive at age 2 months. If we start with one newborn pair, how many pairs of rabbits will we have in the nth month?

2.3 a) Show that the answer to the above question is f_n, where $\{f_n\}$ is the Fibonacci sequence defined in class, or on page 695 of the textbook.

2.3 b) Let $a_n = f_{n+1}/f_n$. Show that $a_{n-1} = 1 + 1/a_{n-2}$. Assuming a_n is convergent find it's limit.