Homework 5 2/13: MATH 112-1 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed. Collaboration with classmates is allowed, but all work submitted must be written out and explained by you.

2 Homework 5 Problems: Series

2.1 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

2.1 a) (11.2.38) $\sum_{k=0}^{\infty} (\sqrt{2})^{-k}$

2.1 d) (11.2.31) $\sum_{n=0}^{\infty} 3^{n+1} 4^{-n}$

2.1 b) (11.2.21) $\sum_{k=1}^{\infty} 12(0.73)^{n-1}$

2.1 e) (11.2.34) $\sum_{k=0}^{\infty} \frac{2^n + 4^n}{e^n}$

2.1 c) (11.2.28)

$$\frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{729} + \cdots$$

2.1 f) (11.2.17)

$$3 - 4 + \frac{16}{3} - \frac{64}{9} + \frac{256}{27} - \frac{1024}{81} + \cdots$$

2.2 (11.2.49) Let $x = 0.999999\ldots$.

2.2 a) Do you think $x < 1$, or $x = 1$?

2.2 b) Use the strategy that $0.213 = \frac{2}{10} + \frac{1}{100} + \frac{3}{1000}$ to use a geometric series to sum x.

2.2 c) How many decimal representations does the number 1 have? Which numbers have more than one decimal representation?

2.3 (11.2.13 modified) Let $a_n = \frac{1}{n^2 + 1}$

2.3 a) Determine whether $\{a_n\}$ is convergent.

2.3 b) Determine whether $\sum_{k=1}^{\infty} a_n$ is convergent.

Extra Problems 2/13: MATH 112-1 Prof. Maxwell Auerbach

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3   Extra Problems: Sequences

3.1 Determine whether the following sequence converges or diverges. If it converges, find the limit. Make sure to show all work, and cite any results from class used.

3.1 a) (original) $a_n = \dfrac{\ln(x)}{e^{2x} - x}$

3.1 d) (original) $\left\{\dfrac{1}{2n}\right\}$

3.1 b) (original) $\{n \ln(n) 2^{-n}\}$

3.1 e) (original) $a_n = \dfrac{1}{2^n}$

3.1 c) (original) $\left\{\dfrac{(-1)^n n^3}{n^2 + 3 - n}\right\}$

3.1 f) (original) $a_n = \dfrac{2n + 5}{n^5 - 3n^3 - 2n^n + n + 1}$

4   Extra Problems: Series

4.1 Determine whether the following series converges or diverges. If it converges, find the value. Make sure to show all work, and cite any results from class used.

4.1 a) (11.2.29) $\displaystyle\sum_{n=1}^{\infty} \dfrac{2 + n}{1 - 2n}$

4.1 c) (original) $\displaystyle\sum_{k=1}^{\infty} k$

4.1 b) (original) $\displaystyle\sum_{k=1}^{\infty} \dfrac{1}{k} - \dfrac{1}{k^3}$

4.1 d) (11.2.29) $\displaystyle\sum_{k=1}^{\infty} e^n$

4.2 (original) If $\displaystyle\sum_{k=1}^{\infty} a_k$ diverges and $c \neq 0$, show that $\displaystyle\sum_{k=1}^{\infty} c\, a_k$ diverges

4.3 (11.2.74) A certain ball has the property that each time it falls from a height h onto a hard, level surface, it rebounds to a height rh, where $0 < r < 1$. Suppose that the ball is dropped from an initial height of H meters.

4.3 a) Assuming that the ball continues to bounce indefinitely, find the total distance that it travels.

4.3 b) Calculate the total time that the ball travels. (Use the fact that the ball falls $gt^2/2$ meters in t seconds.)

4.3 c) Suppose that each time the ball strikes the surface with velocity v it rebounds with velocity $-kv$, where $0 < k < 1$. How long will it take for the ball to come to rest?

4.4 Find the values of x for which the series converges. Find the sum of the series for those values of x.

4.4 a) (11.2.58) $\displaystyle\sum_{n=1}^{\infty} (x + 2)^n$

4.4 c) (11.2.59) $\displaystyle\sum_{n=1}^{\infty} \dfrac{(x - 2)^n}{3^n}$

4.4 b) (11.2.63) $\displaystyle\sum_{n=0}^{\infty} e^{nx}$

4.4 d) (11.2.62) $\displaystyle\sum_{n=1}^{\infty} \dfrac{\sin^n(x)}{3^n}$