Homework 6 2/18: MATH 112-1 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed. Collaboration with classmates is allowed, but all work submitted must be written out and explained by you.

1 Homework 6 Problems: Integral Test

1.1 Determine whether the series is convergent or divergent. If you use a theorem please make sure all assumptions are met.

1.1 a) (11.3.3) $\sum_{k=1}^{\infty} k^{-3}$

1.1 d) (11.3.22) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^2}$

1.1 b) (11.3.25) $\sum_{n=1}^{\infty} \frac{1}{n^2+n^3}$

1.1 e) (11.3.6) $\sum_{k=1}^{\infty} \frac{1}{(3k-1)^4}$

1.1 c) (11.3.14)
$$1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\frac{1}{5\sqrt{5}}$$

1.1 f) (11.3.11)
$$1+\frac{1}{8}+\frac{1}{27}+\frac{1}{64}+\frac{1}{125}$$

1.2 (11.2.13 modified) Determine whether the series given by

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

is convergent using the integral test

1.3 (11.3.33) The Riemann zeta-function ζ is defined by

$$\zeta(x)=\sum_{n=1}^{\infty} \frac{1}{n^x}$$

and is used in number theory to study the distribution of prime numbers. What is the domain of ζ?

Extra Problems 2/18: MATH 112-1 Prof. Maxwell Auerbach

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2 Extra Problems: Integral Test

2.1 Determine whether the series is convergent or divergent. If you use a theorem please make sure all assumptions are met.

2.1 a) (11.3.10) $\sum_{n=3}^{\infty} n^{-0.99999}$

2.1 b) (11.3.21) $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$

2.1 c) (11.3.18) $\sum_{n=1}^{\infty} \frac{1}{n^2+2n+2}$

2.1 d) (11.3.23) $\sum_{n=1}^{\infty} ke^{-k}$

2.1 e) (11.3.24) $\sum_{n=1}^{\infty} ke^{-k^2}$

2.1 f) (11.3.16) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^{3/2}}$

2.1 g) (11.3.5) $\sum_{n=1}^{\infty} \frac{2}{5n-1}$

2.1 h) (original) $\sum_{n=1}^{\infty} \frac{5x^2+10x+3}{(x+1)(2x+1)(x+2)}$

2.2 (11.3.34) Leonhard Euler was able to calculate the exact sum of the p-series with $p = 2$:

$$\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

Use this fact to find the sum of each series.

2.1 a) $\sum_{n=2}^{\infty} \frac{1}{n^2}$

2.1 b) $\sum_{n=3}^{\infty} \frac{1}{(n+1)^2}$

2.1 c) $\sum_{n=1}^{\infty} \frac{1}{(2n)^2}$