Homework 6 2/20: MATH 112-1 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed. Collaboration with classmates is allowed, but all work submitted must be written out and explained by you.

2 Homework 6 Problems: The Comparison Tests

2.1 Determine whether the following series converges or diverges.

2.1 a) (11.4.6) $\sum_{n=1}^{\infty} \frac{n-1}{n^3+1}$

2.1 b) (11.4.7) $\sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$

2.1 c) (11.4.19) $\sum_{n=1}^{\infty} \frac{n+1}{n^3+n}$

2.1 d) (11.4.8) $\sum_{n=1}^{\infty} \frac{6^n}{5^n-1}$

2.1 e) (11.4.10) $\sum_{k=1}^{\infty} \frac{k\sin^2(k)}{1+k^3}$

2.1 f) (11.4.14) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{3n^4+1}}$

2.1 g) (11.4.9) $\sum_{k=1}^{\infty} \frac{\ln(k)}{k}$

2.1 h) (11.4.23) $\sum_{n=1}^{\infty} \frac{5+2n}{(1+n^2)^2}$

2.1 i) (11.4.16) $\sum_{n=1}^{\infty} \frac{1}{n^n}$

2.1 j) (11.4.11) $\sum_{k=1}^{\infty} \frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}}$

2.2 (11.4.39 modified) Let $\sum a_n$ be a convergent series.

2.2 a) Show that if $1 > a_n \geq 0$ then $\sum a_n^2$ also converges.

2.2 b) Use the fact that $\lim_{n\to\infty} a_n = 0$ and tail converges to show that if $a_n \geq 0$ then $\sum a_n^2$ converges.

2.3 (11.4.37) The meaning of the decimal representation of a number $0.d_1d_2d_3\ldots$(where the digit is one of the numbers $0, 1, 2, \ldots, 9$) is that

$$0.d_1d_2d_3d_4\ldots = \frac{d_1}{10} + \frac{d_2}{10^2} + \frac{d_3}{10^3} + \frac{d_4}{10^4} + \cdots$$

Show that this series always converges.

Extra Problems 2/20: MATH 112-1 Prof. Maxwell Auerbach

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3 Extra Problems: Comparison Tests

3.1 Determine whether the following series converges or diverges.

3.1 a) (11.4.29) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

3.1 b) (original) $\displaystyle\sum_{k=3}^{\infty} \frac{\sqrt[4]{k^7+4k+1}}{\sqrt[3]{k^7+4k-1}}$

3.1 c) (11.4.7) $\displaystyle\sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$

3.1 d) (11.4.30) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

3.1 e) (original) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2+1}{k^5+2k^3+3k-1}$

3.1 f) (original) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n+5}{2^n-2}$

3.1 g) (11.4.31) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

3.1 h) (original) $\displaystyle\sum_{n=1}^{\infty} \frac{16+n^2+2n^3-5n^4}{\sqrt[3]{1+n^5+n^9+4n^{11}}}$

3.1 i) (original) $\displaystyle\sum_{k=4}^{\infty} e^{-k}(k+2)(k-3)^{-2}$

3.1 j) (11.4.32) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

3.2 (11.4.40 a) Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is convergent. Prove that if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0$$

then $\sum a_n$ is convergent

3.3 (11.4.40 b) Use 4.2 to show that the following series converge

3.4 a) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$

3.4 b) $\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}\, e^n}$

3.4 (11.4.42) Give an example of a pair of series $\sum a_n$ and $\sum b_n$ with positive terms where $\lim_{n\to\infty}(a_n/b_n) = 0$ and $\sum b_n$ diverges, but $\sum a_n$ converges.