Homework 11 03/25: MATH 112 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed. Collaboration with classmates is allowed, but all work submitted must be written out and explained by you.

1 Homework 11 Problems: Taylor and Maclaurin Series

1.1 Use the definition of a Taylor series to find the Taylor series for $f(x)$ centered at the given value of a. Also find the assoociated radius of convergence.

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1.1 a) (11.10.22) $f(x) = \frac{1}{x}, \quad a = -3$

1.1 b) (11.10.14) $f(x) = e^{-2x}, \quad a = 0$

1.1 c) (11.10.15) $f(x) = 2^x, \quad a = 0$

1.1 d) (11.10.20) $f(x) = x^6 - x^4 + 2, \quad a = -2$

1.2 (11.10.3) If $f^{(n)}(0) = (n+1)!$ for all $n \geq 0$ find the Maclaurin series for f and its radius of convergence

1.3 (11.10.81) Show that if p is an nth-degree polynomial, then

$$p(x+1) = \sum_{k=0}^{n} \frac{p^{(k)}(x)}{k!}$$

1.4 Find the Taylor series for $f(x)$ centered at the given value of a.

2.1 a) (11.10.14) $f(x) = e^{-2x}, \quad a = 0$

2.1 b) (11.10.24) $f(x) = \cos(x), \quad a = \pi/2$

Extra Problems 03/25: MATH 112 Prof. Maxwell Auerbach

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2 Extra Problems: Taylor and Maclaurin Series

2.1 Find the Taylor series for $f(x)$ centered at the given value of a.

2.1 a) (original) $f(x) = x^6 - 2x^5 + 6x, \quad a = -1$

2.1 b) (11.10.26) $f(x) = \sqrt{x}, \quad a = 16$

2.1 c) (original) $f(x) = \dfrac{6}{x^2 - 4}, \quad a = 1$

2.1 d) (11.10.10) $f(x) = \cos^2(x), \quad a = 0$

2.1 e) (11.10.19) $f(x) = x^5 + 2x^3 + x, \quad a = 2$

2.1 f) (11.10.25) $f(x) = \sin(x), \quad a = \pi$

2.1 g) (original) $f(x) = e^{x^2}, \quad a = 0$

2.1 h) (original) $f(x) = \dfrac{1}{2 - 3x}, \quad a = 0$

2.1 i) (11.10.23) $f(x) = e^{2x}, \quad a = 3$

2.1 j) (original) $f(x) = x^2 \cos(2x), \quad a = \pi/4$

2.2 (11.10.84) Show that the function defined by

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is not equal to its Maclaurin series. (You may graph the function afterwards to observe its behavior near the origin).

2.3 Evaluate the indefinite integral as an infinite series. (You may use the series we stated in class for $\cos(x)$ and $\sin(x)$)

2.3 a) (11.10.53) $\displaystyle\int \frac{\cos(x) - 1}{x}\, dx$

2.3 b) (11.10.54) $\displaystyle\int x^2 \sin(x^2)\, dx$