Homework 12 04/01: MATH 112 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed. Collaboration with classmates is allowed, but all work submitted must be written out and explained by you.

1 Homework 12 Problems: Seperable ODEs

1.1 Solve the differential equation.

1.1 a) (9.3.2) $\frac{dy}{dx} = x\sqrt{y}$

1.1 b) (9.3.9) $\frac{dp}{dt} = t^2p - p + t^2 - 1$

1.1 c) (9.3.4) $y' = xe^y$

1.1 d) (9.3.10) $\frac{dz}{dt} + e^{t+z} = 0$

1.2 Find the solution of the differential equation that satisfies the given initial condition.

1.2 a) (9.3.11) $\frac{dy}{dx} = xe^y, \quad y(0) = 0$

1.2 b) (9.3.12) $\frac{dy}{dx} = \frac{x\sin(x)}{y}, \quad y(0) = -1$

1.2 c) (9.3.14) $x + 3y^2\sqrt{x^2+1}\frac{dy}{dx} = 0, \quad y(0) = 1$

1.2 d) (9.3.18) $\frac{dL}{dt} = kL^2\ln(t), \quad L(1) = 1$

1.3 (9.3.21) Solve the differential equation $y' = x + y$ by making the change of variable $u = x + y$.

1.4 (9.3.44) A certain small country has \$10 billion in paper currency in circulation, and over the course of a day \$50 million comes into the country's banks. The government decides to introduce new currency by having the banks replace old bills with new ones whenever old currency comes into the banks. Let $x = x(t)$ denote the amount of new currency in circulation at time t, with $x(0) = 0$.

1.4 a) Formulate a mathematical model in the form of an initial- value problem that represents the "flow" of the new currency into circulation.

1.4 b) Solve the initial-value problem found in part (a).

1.4 c) How long will it take for the new bills to account for 90% of the currency in circulation?

Extra Problems 04/01: MATH 112 Prof. Maxwell Auerbach

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2 Extra Problems: Seperable ODEs

2.1 Solve the differential equation.

2.1 a) (9.3.8) $\frac{dH}{dR} = \frac{RH^2\sqrt{1+R^2}}{\ln(2)}$

2.1 b) (original) $\frac{dx}{dt} = \frac{x\ln(x)}{t}$

2.1 c) (9.3.3) $xyy' = x^2 + 1$

2.1 d) (9.3.6) $\frac{du}{dt} = \frac{1+t^4}{ut^2 + u^4t^2}$

2.1 e) (original) $y' = -y\ln(y/2)$

2.1 f) (9.3.5) $(e^y - 1)y' = 2 + \cos(x)$

2.2 Find the solution of the differential equation that satisfies the given initial condition.

2.2 a) (9.3.16) $\frac{dyP}{dt} = \sqrt{Pt}$,
$P(1) = 2$

2.2 b) (9.3.15) $x\ln(x) = y(1 + \sqrt{3+y^2})y'$,
$y(1) = 1$

2.2 c) (9.3.13) $\frac{du}{dt} = \frac{2t + \sec^2(t)}{2u}$,
$u(0) = -5$

2.2 d) (9.3.17) $y'\tan(x) = a + y$,
$y(\pi/3) = a$. $0 < x < \pi/2$

2.2 e) (original) $\frac{dP}{dt} = P + 4$,
$P(0) = 100$

2.2 f) (original) $\frac{dy}{dt} = \frac{y}{3+t}$,
$y(0) = 1$

2.2 g) (original) $\frac{dz}{dt} = te^z$,
$z(0) = 0$

2.2 h) (original) $\frac{dw}{\theta} = \theta w^2 \sin(\theta^2)$,
$w(0) = 1$

2.2 i) (original) $\frac{dy}{dx} = \frac{5y}{x}$,
$y(1) = 3$

2.2 j) (original) $\frac{dz}{dt} = z + zt^2$,
$z(0) = 5$