Homework 14 4/15: MATH 112 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed.
Collaboration with classmates is allowed, but all work submitted must be written out and explained by you.

1 Homework 14 Problems: Probability Density Functions

1.1 (8.5.3) Let $f(x) = 30x^2(1-x)^2$ for $0 \le x \le 1$ and $f(x) = 0$ for all other x.

1.1 a) Verify that f is a probability density function.

1.1 b) Find $\mathbb{P}(X \le 1/3)$.

1.2 (8.5.4) The density function $f(x) = \dfrac{e^{3-x}}{(1+e^{3-x})^2}$ is an example of a logistic distribution.

1.2 a) Verify that f is a probability density function.

1.2 b) Find $\mathbb{P}(3 \le X \le 4)$.

1.3 (8.5.5) Let $f(x) = \dfrac{c}{(1+x^2)}$

1.3 a) For what value of c is f a probability desnity function?

1.3 b) For that value of c, find $\mathbb{P}(-1 \le X \le 1)$.

1.4 (8.5.6) Let $f(x) = \begin{cases} k(3x - x^2) & \text{if } 0 \le x \le 3 \\ 0 & \text{if } x < 0 \text{ or } 3 < x \end{cases}$.

1.4 a) For what value of k is f a probability density function.

1.4 b) For that value of k, find $\mathbb{P}(X > 1)$.

Extra Problems 4/15: MATH 112-2 Prof. Maxwell Auerbach
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2 Extra Problems: Probability Density Functions

2.1 (original) After measuring the duration of many telephone calls, a telephone company found their data was well approximated by the density function $p(x) = 0.4e^{-0.4x}$, where x is the duration of a call, in minutes.

2.1 a) What percentage of calls last between 1 and 2 minutes?

2.1 b) What percentage of calls last 1 minute or less?

2.1 c) What percentage of calls last 3 minutes or more?

2.2 (original) Give an example of the following:

2.2 a) A density function that is greater than zero on $0 \le x \le 20$ and zero everywhere else, and is strictly increasing.

2.2 c) A probability density function which is nonzero only between $x = 2$ and $x = 7$ that is distinct from your answer in a) and b).

2.2 b) A probability density function which is nonzero only between $x = 1$ and $x = 5$, and is strictly decreasing.

2.2 d) A probability density function which is positive between $x = a$ and $x = b$ and zero everywhere else.

2.3 (8.5.12) The time between infection and the display of symptoms for streptococcal sore throat is a random variable whose probabililty density function can be approximated as a function of t hours by

$$f(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{15676}t^2 e^{-0.05t} & 0 \le t \le 150 \\ 0 & 150 < t \end{cases}$$

2.3 a) What is the probability that an infected patient will display symptoms within the first 48 hours?

2.3 b) What is the probability that an infected patient will not display symptoms until after 36 hours?

2.4 (8.5.1) Let $f(x)$ be the probability density function for the lifetime of a manufacturer's highest quality car tire, where x is measured in miles. Explain the meaning of each integral.

2.4 a) $\int_{30,000}^{40,000} f(x)\, dx$

2.4 b) $\int_{25,000}^{\infty} f(x)\, dx$