Homework 14 04/17: MATH 112 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed. Collaboration with classmates is allowed, but all work submitted must be written out and explained by you.

2 Homework 14 Problems: Mean, Median, and Standard Deviation

2.1 (8.5.4 modified) The density function

$$f(x) = \frac{e^{3-x}}{(1+e^{3-x})^2}$$

is an example of a logistic distribution

2.1 a) Verify that f is a probability distribution.

2.1 b) Find the median of f

2.1 c) Find $\mathbb{P}(3 \leq X \leq 4)$.

2.2 (8.5.13 modified) REM sleep is the phase of sleep when most active dreaming occurs. In a study, the amount of REM sleep during the first four hours of sleep was described by a random variable T with probability density function

$$f(t) = \begin{cases} \frac{1}{1600}t & \text{if } 0 \leq t \leq 40 \\ \frac{1}{20} - \frac{1}{1600}t & \text{if } 40 < t \leq 80 \\ 0 & \text{if } t < 0 \text{ or } t > 80 \end{cases}$$

2.2 a) Show that $f(t)$ is a probability density function

2.2 b) What is the probability that the amount of REM sleep is between 30 and 60 minutes?

2.2 c) Find the expected amount of REM sleep.

2.3 (original) A quantity x is distributed with density function $p(x) = (2-x)/2$ for $0 \leq x \leq 2$ and $p(x) = 0$ otherwise.Find the mean and median of x.

2.4 (original) Suppose that t measures the time (in hours) it takes for a student to complete an exam. All students are done within two hours and the density function for t is

$$p(t) = \begin{cases} t^3/4 & \text{if } 0 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

2.4 a) What proportion of students take between 1.5 and 2.0 hours to finish the exam?

2.4 b) What is the mean time for students to complete the exam?

2.4 c) Compute the standard deviation of this distribution.

Extra Problems 04/17: MATH 112-1 Prof. Maxwell Auerbach

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3 Extra Problems: Mean, Median, and Standard Deviation

3.1 (8.5.21) The hydrogen atom is composed of one proton in the nucleus and one electron, which moves about the nucleus. In the quantum theory of atomic structure, it is assumed that the electron does not move in a well-defined orbit. Instead, it occupies a state known as an orbital, which may be thought of as a "cloud" of negative charge surrounding the nucleus. At the state of lowest energy, called the ground state, or 1s-orbital, the shape of this cloud is assumed to be a sphere centered at the nucleus. This sphere is described in terms of the probability density function

$$p(r) = \frac{4}{a_0^3} r^2 e^{-2r/a_0}, \quad r \geq 0$$

where a0 is the *Bohr radius* which we may think of as a fixed constant (in actuality, we know that $a_0 \approx 5.59 \times 10^{-11}$m). The cumulative distribution function of the distance, r given by the integral

$$P(r) = \int_0^r \frac{4}{a_0^3} s^2 e^{-2s/a_0}\, ds$$

gives the probability that the electron will be found within the sphere of radius r meters centered at the nucleus.

3.1 a) Verify that $p(r)$ is a density function.

3.1 b) Find $\lim_{r\to\infty} p(r)$. For what value of r does $p(r)$ have its maximum value.

3.1 c) Graph the density function using any means you wish.

3.1 d) Find the probability that the electron will be within the sphere of radius $4a_0$ centered at the nucleus.

3.1 e) Calculate the mean distance of the electron from the nucleus in the ground state of the hydrogen atom.

3.1 f) Calculate the median distance of the electron from the nucleus in the ground state of the hydrogen atom.

3.2 (8.5.10) A type of light bulb is labeled as having an average lifetime of 1000 hours. It's reasonable to model the probability of failure of these bulbs by an exponential density function with mean $\mu = 1000$.

3.1 a) Use this model to find the probability that a bulb fails within the first 200 hours.

3.1 b) Use this model to find the probability that a bulb burns for more than 800 hours.

3.1 c) What is the median lifetime of these light bulbs?