

Homework 12 04/03: MATH 112 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed. Collaboration with classmates is allowed, but all work submitted must be written out and explained by you.

2 Homework 12 Problems: Integrating Factor for Linear ODEs

2.1 Solve the differential equation.

2.1 a) (9.5.6) $y' - y = e^x$

2.1 c) (9.5.13) $t^2 y' + 3ty = \sqrt{1+t^2}, \quad t > 0$

2.1 b) (9.5.10) $2xy' + y = 2\sqrt{x}$

2.1 d) (9.5.12) $y' + 2xy = 1$

2.2 Solve the initial-value problem.

2.2 a) (9.5.18) $xy' + y = x \ln(x), \quad y(1) = 0$

2.2 c) (9.5.17) $tu' = t^2 + 3u, \quad t > 0, u(2) = 4$

2.2 b) (9.5.16) $t^3 y' + 3t^2 y = \cos(t), \quad y(\pi) = 0$

2.2 d) (9.5.15) $x^2 y' + 2xy = \ln(x), \quad y(1) = 2$

2.3 (9.5.31) Let $P(t)$ be the performance level of someone learning a skill as a function of the training time t . The graph of P is called a learning curve. In Exercise 9.1.15 the book proposed the differential equation

$$\frac{dP}{dt} = k(M - P(t))$$

as a reasonable model for learning where k is a positive constant.

2.3 a) Solve the above as a linear differential equation.

2.3 b) Use your solution to sketch graph the learning curve (you may for once use an outside calculator or online tool to help you here).

Extra Problems 04/03: MATH 112 Prof. Maxwell Auerbach

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3 Extra Problems: Integrating Factor for Linear ODEs

3.1 Solve the following differential equation or initial value problem.

3.1 a) (9.5.9) $xy' + y = \sqrt{x}$

3.1 g) (9.5.5) $y' + y = 1$

3.1 b) (9.5.14) $t \ln(t) \frac{dr}{dt} + r = te^t$

3.1 h) (9.5.20) $(x^2 + 1) \frac{dy}{dx} + 3x(y - 1) = 0, \quad y(0) = 2$

3.1 c) (original) $(x^2 + 1)y' = xy - 3, \quad y(0) = 4$

3.1 i) (original) $y' + y = \cos(x), \quad y(0) = 1$

3.1 d) (9.5.7) $y' = x - y$

3.1 j) (9.5.8) $4x^3y + x^4y' = \sin^3(x)$

3.1 e) (9.5.11) $xy' - 2y = x^2, \quad x > 0$

3.1 k) (original) $y' - 4\frac{1}{x}y = x^3, \quad y(1) = 2$

3.1 f) (original) $x^3y' = -yx^2 - x^5, \quad y(2) = 0$

3.1 l) (9.5.19) $xy' = y + x^2 \sin(x), \quad y(\pi) = 0$

3.2 (9.5.26) Solve the second-order equation

$$xy'' + 2y' = 12x^2$$

by making the substitution $u = y'$.

3.3 (9.5.34) A tank with a capacity of 400 L is full of a mixture of water and chlorine with a concentration of 0.05 g of chlorine per liter. In order to reduce the concentration of chlorine, fresh water is pumped into the tank at a rate of 4 L/s. The mixture is kept stirred and is pumped out at a rate of 10 L/s. Find the amount of chlorine in the tank as a function of time.