Extra Problems 4/08: MATH 112 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed. Collaboration with classmates is allowed, but all work submitted must be written out and explained by you.

1 Extra Problems: Taylor Polynomial Applications

1.1 For each of the following first approximate f by a Taylor polynomial with degree n at the number a, then use Taylor's Inequality to estimate the accuracy of the approximation $f(x) < T_n(x)$ when x lies in the given interval.

1.1 a) (11.11.13) $f(x) = 1/x, \quad a = 1, \quad n = 2 \;\; 0.7 \leq x \leq 1.3$

1.1 b) (11.11.20) $f(x) = x\ln(x), \quad a = 1, \quad n = 3 \;\; 0.5 \leq x \leq 1.5$

1.1 c) (11.11.16) $f(x) = \sin(x), \quad a = \pi/6, \quad n = 4 \;\; 0 \leq x \leq \pi/3$

1.1 d) (11.11.19) $f(x) = e^{x^2}, \quad a = 0, \quad n = 3 \;\; 0 \leq x \leq 0.1$

1.1 e) (11.11.14) $f(x) = x^{-1/2}, \quad a = 4, \quad n = 2 \;\; 3.5 \leq x \leq 4.5$

1.2 (11.11.26) How many terms of the Maclaurin series for $ln(1 + x)$ do you need to use to estimate $ln(1.4)$ to within 0.001?

1.3 (11.11.31) A car is moving with speed 20 m/s and acceleration 2 m/s^2 at a given instant. Using a second-degree Taylor polynomial, estimate how far the car moves in the next second. Would it be reasonable to use this polynomial to estimate the distance traveled during the next minute?

Extra Problems: MATH 112-2 Prof. Maxwell Auerbach

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2 Extra Problems: Taylor Polynomial Applications

2.1 For each of the following first approximate f by a Taylor polynomial with degree n at the number a, then use Taylor's Inequality to estimate the accuracy of the approximation $f(x) < T_n(x)$ when x lies in the given interval.

2.1 a) (11.11.8+) $f(x) = x\cos(x), \quad a = 0, \quad n = 3 \quad -\pi/2 \leq x \leq \pi/2$

2.1 b) (11.11.4+) $f(x) = \sin(x), \quad a = \pi/6, \quad n = 4 \quad -7\pi/6 \leq x \leq 9\pi/6$

2.1 c) (11.11.18) $f(x) = \ln(1 + 2x), \quad a = 1, \quad n = 3 \;\; 0.5 \leq x \leq 1.5$

2.1 d) (11.11.9+) $f(x) = xe^{-2x}, \quad a = 0, \quad n = 3 \quad -.1 \leq x \leq .1$

2.1 e) (11.11.21) $f(x) = x\sin(x), \quad a = 0, \quad n = 5 \quad -1 \leq x \leq 1$

2.1 f) (11.11.15) $f(x) = x^{2/3}, \quad a = 1, \quad n = 3 \;\; 0.8 \leq x \leq 1.2$

2.1 g) (11.11.5+) $f(x) = \cos(x), \quad a = \pi/2, \quad n = 5 \;\; 0 \leq x \leq pi$

2.1 h) (11.11.3+) $f(x) = e^x, \quad a = 1, \quad n = 4 \;\; .8 \leq x \leq 1.2$

2.1 i) (11.11.10+) $f(x) = \arctan(x), \quad a = 1, \quad n = 3 \;\; 0.9 \leq x \leq 1.1$

2.2 (11.11.33) An electric dipole consists of two electric charges of equal magnitude and opposite sign. If the charges are q and $-q$ and are located at a distance d from each other, then the electric field E at the point P in the figure is

$$E = \frac{q}{D^2} - \frac{q}{(D+d)^2}$$

By expanding this expression for E as a series in powers of d/D, show that E is approximately proportional to $1/D^3$ when P is far away from the dipole.