INVERSE SPECTRAL THEORY WORKSHOP AT EMORY UNIVERSITY APRIL 24–26, 2026

Abstracts in alphabetical order (by speaker's last name)

Roman Bessonov, *From local to global asymptotics of orthogonal polynomials*

Let $\{\phi_n^*\}$ be the sequence of reflected orthogonal polynomials on the unit circle $\mathbb{T}$ generated by a measure μ of Szegő class, and let D_μ be the Szegő function of μ. We prove the uniform Cesàro asymptotics

$$\sup_{z\in\Gamma_\zeta}\left(\frac{1}{n}\sum_{k=1}^{n}\left||\phi_k^*(z)D_\mu(z)|^2-1\right|\right)\to 0, \qquad n\to\infty,$$

for almost all Stolz angles Γ_ζ. This extends the well-known asymptotic result of Máté, Nevai, and Totik (1991) from the local scale $O(1/n)$ near $\mathbb{T}$ to the global scale $O(1)$. We also study asymptotic behaviour of arguments of orthogonal polynomials and extend some classical results due to Szegő using a new technique. Joint work with Artur Nicolau (Universitat Autònoma de Barcelona).

David Damanik, *The Deift Conjecture and Generalized Almost Periodicity I*

This talk, and the talk by Long Li, will present our joint work with Nicolae Strungaru on a new perspective on the Deift conjecture. This conjecture states that the Korteweg-de Vries (KdV) equation with almost periodic initial data admits global solutions that are almost periodic in space and time. There has been exciting progress on this conjecture in the past couple of decades. The status quo is that it has been confirmed for suitable classes of almost periodic initial data, and it has also been realized that almost periodic initial data may develop discontinuities, which are incompatible with Bochner/Bohr almost periodicity. Our work is motivated by recent work of Daniel Lenz, Timo Spindeler, and Nicolae Strungaru on the characterization of structures with pure point diffraction - the central property in mathematical quasicrystal theory. It had been understood for a long time that Bochner/Bohr almost periodicity is not the right notion in the diffraction context, but Lenz-Spindeler-Strungaru have only recently realized precisely which almost periodicity notions are the correct ones when studying and even characterizing key properties in diffraction theory. Armed with these realizations, we give the Deift conjecture a fresh look.

Sergey Denisov, *The Nonlinear Carleson conjectures and existence of wave operators*

We will discuss the pointwise asymptotics of orthogonal polynomials for measures in the Szego class and relate it to the existence of wave operators for Jacobi matrices. The inverse scattering theory turns out to be a useful tool in our analysis (partially based on the joint project with Giorgio Young).

Benjamin Eichinger, *New Universality classes associated to fractals*

Universality describes the phenomena that the local behavior of zeros of orthogonal polynomial depends only on general characteristics of the system, rather than on its precise properties. From a modern perspective, such local behavior is analyzed through local scaling limits of the Christoffel–Darboux kernel. Recent works have established necessary and sufficient conditions for the most studied universality classes, including bulk universality and hard-edge universality. In all these settings, the rescaled Christoffel–Darboux kernel converges to a single limit kernel.

In this talk, we consider scaling limits in which there is not a single limit kernel, but rather an entire cycle of limit kernels. We show that this type of behavior arises naturally when studying scaling limits for orthogonal polynomials associated with the equilibrium measure of the real Julia set of an expanding polynomial or with the Cantor measure on the classical middle-third Cantor set.

This talk is based on joint work with Alexander Kheifets, Milivoje Lukić, and Peter Yuditskii.

Fritz Gesztesy, *Essential self-adjointness for a class of pseudo-differential operators perturbed by strongly singular potential coefficients*

For $\emptyset \neq X \subset \mathbb{R}^n$ discrete, $n \in \mathbb{N}$, we characterize the distributions in $\mathcal{D}'(\mathbb{R}^n)$ with support contained in X and subsequently use this result to prove that $C_0^\infty(\mathbb{R}^n \backslash X)$ is dense in the fractional Sobolev space $H^s(\mathbb{R}^n)$ if and only if $s \in (-\infty, n/2]$. This fact is then used to show that

$$\begin{gathered}(-\Delta)^s\big|_{C_0^\infty(\mathbb{R}^n\backslash X)} \text{ is essentially self-adjoint in } L^2(\mathbb{R}^n)\\ \text{if and only if } s \in (0, n/4] \text{ (i.e., if and only if } n \geq 4s),\end{gathered}$$

and analogously for the operator $\big(-\Delta + m^2 I\big)^s\big|_{C_0^\infty(\mathbb{R}^n\backslash X)}$, $m \in (0,\infty)$.

In addition, for $n \in \mathbb{N}$, $s \in (0, n/4)$, applying the fractional Birman–Hardy–Rellich-type inequality combined with Kato–Rellich–Wüst perturbation theory, we prove essential self-adjointness (resp., self-adjointness) of

$$\big[(-\Delta)^s + c|\,\cdot\,|^{-2s}\big]\big|_{H^{2s}(\mathbb{R}^n)}$$

in $L^2(\mathbb{R}^n)$ if $c \in [-C_{n,s}, C_{n,s}]$ (resp., $c \in (-C_{n,s}, C_{n,s})$). Here $C_{n,s}$ is given by

$$C_{n,s} = 2^{2s}\Gamma((n/4) + s)/\Gamma((n/4) - s), \quad s \in (0, n/4).$$

As an introduction into this subject we will also discuss the case of even-order, strongly singular ODE operators on the half-line $(0, \infty)$.

This talk is based on joint work with Markus Hunziker, Dorina Mitrea (Baylor Univ., TX, USA) and Gerald Teschl (Univ. of Vienna, Austria).

Burak Hatinoglu, *Mixed data generalizations of Borg–Levinson and Marchenko theorems*

We consider inverse spectral problems for one-dimensional Schrödinger operators on a finite interval with integrable potentials. Classical results of Borg and Levinson show that the potential can be uniquely recovered from two spectra, while Marchenko's theorem establishes uniqueness from the spectral measure (equivalently, the Weyl m-function).

In this talk, after a brief review of these foundational results, we discuss a general framework for inverse spectral problems with mixed data of Borg-Levinson and Marchenko settings. Motivated by the principle that one spectrum provides "half" of the spectral information, we address the question of whether the remaining information can be recovered from partial data distributed across different sources. In particular, we consider that one full spectrum, together with subsets of another spectrum and subsets of norming constants, suffices to uniquely determine the potential, even when the missing data come from non-matching index sets with some convergence conditions.

We then extend this mixed-data perspective beyond the Sturm-Liouville setting to discrete Jacobi operators and to canonical Hamiltonian systems. The latter provides a unifying framework that encompasses a wide class of second-order operators. Our approach is based on complex function theory, in particular the analysis of Weyl m-functions as meromorphic Herglotz functions.

Ilya Kachkovskiy, *Multi-dimensional quasiperiodic operators with monotone potentials*

Abstract: In 1980s, Craig and Pöschel constructed examples of quasiperiodic operators with bounded sawtooth-type monotone potentials that demonstrate pure point spectrum in the perturbative regime. However, these examples were obtained using inverse spectral theory, by the means of fine-tuning the sampling function in such a way that "large numerators" caused by discontinuities do not appear on any step of the direct KAM-type scheme.

In the talk, I will focus on the main ideas of a direct KAM-type scheme which works for all (anti-)Lipschitz monotone potentials, whether or not such large numerators are present, thus providing a perturbative proof of localization, as well as information about the spectral gaps, for a large class of these potentials. The talk is based on a joint work with L. Parnovski and R. Shterenberg.

Long Li, *The Deift Conjecture and Generalized Almost Periodicity II*

In the work of Lenz-Spindeler-Strungaru, they answered the long standing problem of the characterization of pure point diffraction. They showed that the translation bounded measure has a pure point diffraction measure if and only if it is mean almost periodic. Furthermore, they also solve the phase problem and uniform phase problem by identifying that the measure is Besicovitch and Weyl almost periodic respectively. Motivated by their work and the observation that the violation of Bohr almost periodicity in the theory of Sodin-Yuditskii when the direct Cauchy Theorem condition fails is very mild, we would like to investigate if there is still an appropriate notion of almost periodicity in the solution of the KdV equation. It turns out that the notions of generalized almost periodicity and Weyl almost periodicity still make good sense in this setting. I will also discuss the possibility of introducing $\mathbb{R}^2$ almost periodicity to the theory by looking at some examples that are related to the counterexample of the Deift's conjecture. This is based on a joint work with David Damanik and Nicolae Strungaru.

Doron Lubinsky, *Local Limits for Sequences of Polynomials*

Let $\{P_n\}$ be a sequence of polynomials with real zeros. Local limits have the form

$$\lim_{n\to\infty} \frac{P_n\left(\xi_n + \frac{z}{\Lambda_n}\right)}{P_n\left(\xi_n\right)} = f\left(z\right),$$

uniformly for z in compact subsets of the complex plane. Here $\{\xi_n\}$ are appropriate points and $\{\Lambda_n\}$ are appropriate scaling factors. A typical example is provided by Chebyshev polynomials:

Theorem (2025)
Let E be a compact subset of the real line, containing a closed interval $I = [c, d]$ in its interior. Let $\omega_E = \mu_E'$ denote the density of the equilibrium measure μ_E for E. For $n \geq 1$, let $\xi_n \in I$ denote an alternation point of P_n, the Chebyshev polynomial of degree n for E. Then uniformly for z in compact subsets of the plane,

$$\lim_{n\to\infty} \frac{P_n\left(\xi_n + \frac{z}{n\omega_E(\xi_n)}\right)}{P_n\left(\xi_n\right)} = \cos \pi z.$$

As a consequence, one obtains "clock spacing" of the zeros. There are similar limits for orthogonal polynomials. We discuss these also, and their relation to universality limits.

Alexei Rybkin, *Inverse Scattering for KdV with Embedded Bound States*

We study the inverse scattering problem for one-dimensional Schrödinger operators in the presence of embedded bound states and investigate their role in the dynamics of the Korteweg–de Vries (KdV) equation. While the inverse scattering transform is well understood for rapidly decaying or periodic potentials, significantly less is known for slowly decaying initial data, where spectral singularities and embedded eigenvalues may occur.

Focusing on a class of long-range potentials of Wigner–von Neumann type, we develop a framework that extends elements of scattering theory beyond the classical setting. In particular, we introduce appropriate spectral data that incorporate embedded bound states through additional norming constants and show that these states cannot be detected by the reflection coefficient alone.

Our main result provides a constructive transformation that converts resonances into embedded bound states while preserving key scattering features. This approach is closely related to binary Darboux transformations and is naturally adapted to the inverse scattering framework. As a consequence, we obtain explicit families of solutions to the KdV equation exhibiting bounded positon behavior and show that embedded bound states persist under the KdV flow.

These results contribute to the broader program of extending integrable systems techniques to nonstandard classes of initial data and offer new insight into the interplay between spectral theory and nonlinear wave propagation beyond the short-range regime.

Selim Sukhtaiev, *The Maslov index in spectral theory: an overview*

This talk is centered around a symplectic approach to eigenvalue problems for systems of ordinary differential operators (e.g., Sturm-Liouville operators, canonical systems, and quantum graphs), multidimensional elliptic operators on bounded domains, and abstract self-adjoint extensions of symmetric operators in Hilbert spaces. The symplectic view naturally relates spectral counts for self-adjoint problems to the topological invariant called the Maslov index. In this talk, the notion of the Malsov index will be introduced in analytic terms and an overview of recent results on its role in spectral theory will be given.

Giorgio Young, *Optimal dispersion for discrete periodic Schrödinger operators*

In this talk, I will describe recent work proving a dispersive estimate at the sharp rate for discrete periodic Schrödinger operators on $\ell^2(\mathbb{Z})$. After motivating dispersion as a measure of wavefunction spread, I will describe our proof. The main input is an analysis of the Marchenko–Ostrovski mapping. This talk is based on joint work with David Damanik and Jake Fillman.